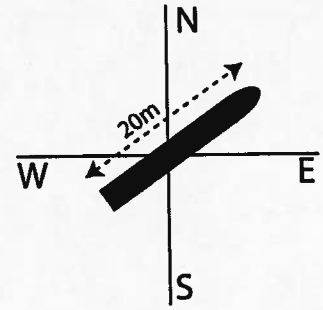
2013 - 45th SIN-Phil* Exam

1) In an unfortunate incident, passengers on a Carousel Cruise Lines ship are confined to the poop deck. The partially-disabled ship is capable of traveling at 8m/s on calm water. Today it has a velocity directed due North, however, it has steered at a certain angle to compensate for a current flowing from the East at 5m/s. A rat runs across the poop deck, of length 20m, parallel to the axis of the ship, at 2m/s. The rat runs from back to front, pauses 10 seconds to rest, and runs front to back at the same speed. What is the length of the rat's path, as traced out over the ocean floor?



(a) 62 m **(b)** 78 m **(c)** 187 m **(d)** 190 m **(e)** 193 m

* This exam is dedicated to the late Professor Phil Eastman, founder of the SIN exam.

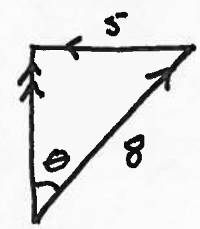
Let $\vec{v}_{S,\omega}$ = Velocity of the ship relative to water = 8 m/s

$\vec{v}_{\omega,E}$ = " " water relative to Earth = 5 m/s ←

$\vec{v}_{S,E}$ = " " the ship relative to Earth = ↑

$\vec{v}_{S,E} = \vec{v}_{S,\omega} + \vec{v}_{\omega,E}$

↑ 8 $\xleftarrow{5}$



$\therefore \theta = \sin^{-1}(5/8)$

$= 38.68°$

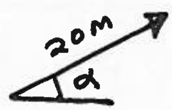
Calculation of the path of the Rat

i) time taken to move from the back to the front. $v = \frac{d}{t}$

$\therefore \quad t = \frac{20}{2} = 10$ sec.

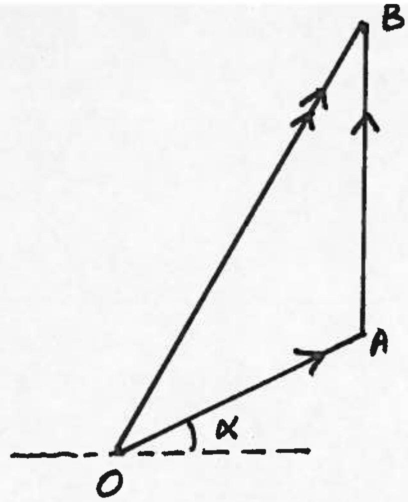
$d_{R,E}$ = displacement of the Rat relative to Earth

$\vec{d}_{R,E} = \vec{d}_{R,S} + \vec{d}_{S,E}$



↑ $= (v_{S,E})\,t = 6.24$ m/s × 10 sec = 62.45 m.

$OA = 20\,m, \quad AB = 62.45\,m$



∴ using the Cosine Rule

$$OB^2 = 20^2 + 62.45^2 - 2(20)(62.45)\cos 141.32$$

$$\therefore OB = \underline{79.05}\,m.$$

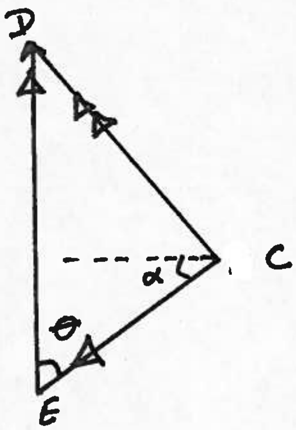
ii) now it rests for 10 sec

∴ the displacement would be $\underline{62.45\,m} \uparrow$

iii) now it moves from the front to the back.

$$d_{R,E} = d_{R,S} + d_{S,E}$$

$CD = ??$ $\quad CE = 20, \; ED = 62.45, \; \theta = 38.68°$



∴ using Cosie Rule

$$CD = \underline{48.47\,m}.$$

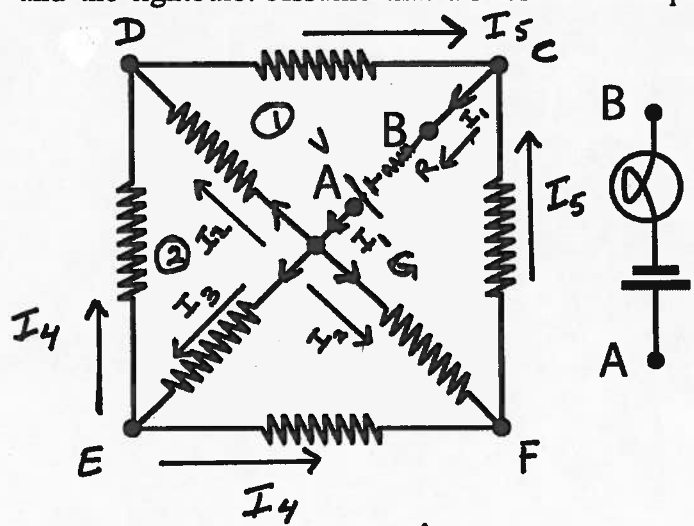
∴ the Total <u>distance</u> moved. or the <u>length</u> of the Rat's path is

$$79.05 + 62.45 + 48.47 = 189.9 \approx \underline{\underline{190\,m}}.$$

<u>answer</u> is d

2) Kalle Anka Scrooge needs to increase his investment in a factory for black hole- powered garbage disposal units. To do this he has an ingenious idea on how to conserve his hard-earned cash, by changing the power delivered to a light bulb. He connects a battery together with a lightbulb that has a 10 ohm internal resistance, between points A and B as shown, and determines the current through the lightbulb. Determine this current, as a factor times the current in a simple circuit consisting of only the battery and the lightbulb. Assume that all resistances equal 10 ohms.

(a) 8/15
(b) 8/7
(c) 1/5
(d) 1/2
(e) 1



Currents are set due to symmetry

At junction "G" using kirchhoff's law

$$I_1 = 2I_2 + I_3$$

at E, $\quad 2I_4 = I_3$

at D, $\quad I_5 = I_4 + I_2$

and at C, $\quad 2I_5 = I_1$

For loop #1

$$I_2 R + I_5 R + I_1 R = V$$

$$R(I_2 + I_1 + I_5) = V = (3I_5 + I_2)R$$

For loop 2 $\quad (I_3 + I_4 - I_2)R = 0$

$$I_3 + I_3/2 - I_2 = 0$$

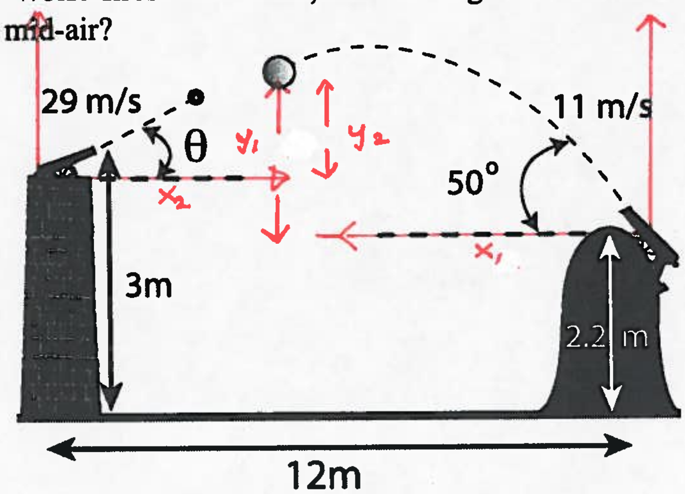
$\therefore I_3 = \frac{2}{3} I_2 \quad$ now $I_1 = 2I_2 + \frac{2}{3} I_2 = \frac{8}{3} I_2.$

$\therefore V = (3I_5 + I_2)R = \left(3\frac{I_1}{2} + \frac{3}{8} I_1\right)R$

$\therefore I_1 = \left(\frac{8}{15}\right)\left(\frac{V}{R}\right) \qquad \therefore I_1 \propto \left(\frac{8}{15}\right)$ answer (a)

3) The Marquis de Montcalm, on top of a 3m high wall, is under attack by Major General Wolfe. Wolfe has a large cannon on a hill that is 12m away and 2.2m high, using large balls for projectiles. Wolfe's cannon fires its balls with a speed of 11m/s, at an angle of 50 degrees above the horizontal. The Marquix de Montcalm has a cannon that shoots small balls, but at a speed of 29 m/s. If Montcalm shoots 0.6 s after Wolfe fires his cannon, at what angle Θ must he fire in order to intercept Wolfe's projectiles in mid-air?

(a) 22.4°
(b) 26.9°
(c) 36.7°
(d) 48.3°
(e) 56.3°



Let "t" be the time taken for the "Collision" relative to "Montcalm."

using $(\bar{x}-\bar{x}_0) = \bar{v}_{ox} t + \frac{1}{2}\bar{a}_x t^2 \longrightarrow +ve$

$a_x = 0$

For Montcalm $x_2 = 29 \cos\theta \, t$

For Wolfe $x_1 = 11 \cos 50 \, (t + 0.6)$

$x_1 + x_2 = 12$

$\therefore$ we can say $29 \cos\theta \, t + 11 \cos 50 (t + 0.6) = 12$

$29 \cos\theta \, t + 7.07 \, t + 4.24 = 12$

Vertical Motion during this time.

use $(y_1 - y_0) = V_{oy} t + \frac{1}{2} a_g t^2$

$y_1 = 11 \sin 50 \, (t + 0.6) - 4.9 \, (t + 0.6)^2$

$y_2 = 29 \sin\theta \, t - 4.9 t^2$

$y_1 - y_2 = 0.8$

$\therefore \; 0.8 = 2.55 \, t + 3.29 - 29 \sin\theta \, t$

We can rewrite this as

$$29 \sin\theta\, t = 2.55\, t + 2.49 \quad \text{— ②}$$
$$29 \cos\theta\, t = 7.76 - 7.07\, t \quad \text{— ①}$$

$①^2 + ②^2$

$$29^2 t^2 = 6.5 t^2 + 12.7 t + 6.2 + 60.2 + 49.9 t^2 - 109.7 t$$

$$784.6\, t^2 + 97 t - 66.4 = 0$$

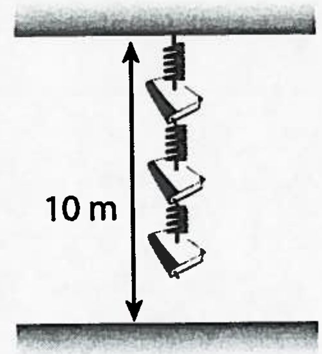
using the positive value of t

$$t = 0.236 \text{ sec.}$$

sub into ② $\Rightarrow$ $\theta = 26.89 \approx \underline{\underline{26.9^\circ}}$

<u>answer</u> (b).

4) A Canadian MP misunderstands the process by which a private member's bill is introduced to the "floor of parliament". The MP attaches a massless spring with a constant k = 100N/m to the ceiling of parliament, and a (very thin) copy of the bill of mass m = 1kg to the end of the spring. He then attaches another spring, exactly the same as the first one, to the other side of the bill. He repeats this, adding copies of the bill and springs until the contraption touches the floor. If the length of an unextended spring is 50cm, and the distance between the ceiling and the floor of parliament is 10m, how many springs does the MP need?



(a) 9 (b) 10 (c) 11 (d) 12 (e) 13

If we look at the last spring, it would extend due to a single mass "m".
The second last spring would extend due to mass 2m and the third due to mass 3m etc.

For the last spring using Hooke's Law

$\sum \bar{F}_{ext} = m\bar{a} \Rightarrow \quad mg - ky_1 = 0 \quad \therefore y_1 = mg/k$

for the 2nd last $\quad 2mg - ky_2 = 0 \quad \therefore y_2 = 2mg/k.$

$\vdots$

each spring has a length of 0.5 m.

$\therefore y_1 + y_2 + y_3 + \cdots y_n + n(0.5) = 10 \quad — ①$

$m = 1kg$ & $k = 100 N/m.$

$\therefore y_1 = \frac{1(9.8)}{100} = 0.098$

$\therefore$ we could rearrange ①

$$\sum_{i=1}^{n} y_i = \frac{10 - n(0.5)}{0.098} = 102.04 - \frac{0.5n}{0.098}$$

We could now use a trial & Error method or realize that

$$\sum_{i=1}^{n} i = \frac{(n+1)n}{2}$$

$$\therefore \frac{n(n+1)}{2} = 102.04 - \frac{0.5n}{0.098}$$

which gives us $n^2 + 11.2n - 204 = 0$

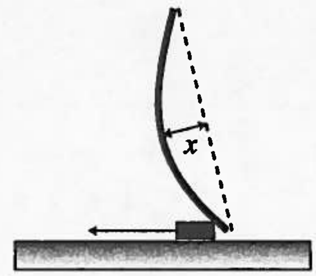
$$\therefore n = \frac{-11.2 \pm \sqrt{(11.2)^2 + 4(204)}}{2}$$

taking the positive value

$$n \simeq 9.7 \simeq \underline{\underline{10}}.$$

answer: (b)

5) A certain power-forward from the Toronto Maple Laughs shoots a 170g puck with a 100 km/hr slap shot. A physicist sitting on the bench observes the 0.33kg hockey stick to bend as it hits the ice, such that it obeys Hooke's law, $F = -kx$, where x = 9cm and k = 3,740N/m. If the blade of the stick is in contact with the puck over a distance d = 30cm, what is the total work done on the puck if it starts from rest? Answer in Joules.



(a) 15.2 **(b)** 65.6 **(c)** 101 **(d)** 116 **(e)** 127

This is more a conceptual question.

Law: The total work done on a system is equal to the change in Kinetic Energy.

$W_T = \Delta KE$

$\therefore$ total work done on the puck $= \left[\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2\right]$

$v_i = 0$ since the puck was at rest at first

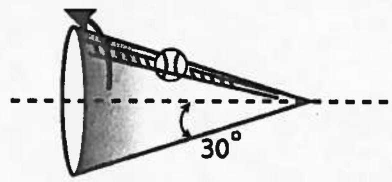
$\therefore \frac{1}{2} m v_f^2 = \left[\frac{1}{2}(0.17)\left[\frac{100 \times 1000}{3600}\right]^2\right] - 0$

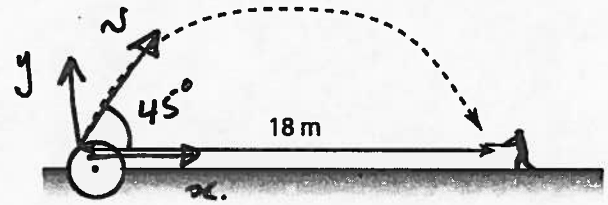
$= 65.6\ J$

answer (b)

6) In a desperate attempt to solve their pitcher problems, the Blue Jays come up with an automated solution. A baseball is launched from the outside surface of a cone rotating around its axis at an angular speed of 400 degrees per second. The cone is horizontal, with its axis at the same height as the ground, and with its lateral surface making an angle of 30 degrees with the horizontal. The baseball is held in a clasp that can slide up and down the outside surface on a track, which can release the ball on command. What minimum horizontal distance down the axis of the cone (measured from its apex) must the ball be released so that it travels 18m horizontally before being hit by the batter? Assume the ball is hit at the same height as it is thrown.

(a) 0.976m
(b) 1.14m
(c) 1.69m
(d) 2.93m
(e) 3.30m





We need to maximize our horizontal distance hence $\theta = 45°$.

$V_{ox} = v\cos 45$
$V_{oy} = v\sin 45$

horizontal displacement: $(x - x_0) = V_{ox}t + \frac{1}{2}a_x t^2$.

$a_x = 0 \quad \therefore \quad 18 = v\cos 45\, t$ — ①

Vertical displacement ↑ up is +ve.

$(y - y_0) = V_{oy}t + \frac{1}{2}a_y t^2$

$0 = v\sin 45\, t - 4.9t^2 \quad \therefore \quad t = \frac{v\sin 45}{4.9}$

sub into ①

$\therefore \quad 18 = \frac{v\cos 45\, v\sin 45}{4.9} = \frac{v^2}{9.8} \quad \therefore \quad v = 13.28$ m/s

$\omega = 6.98$ Rad/sec and $v = R\omega = 13.28$

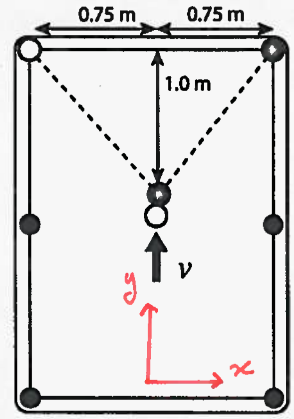
$\therefore R = 1.9$ m.

now $\tan 30 = \frac{1.9}{x} \quad \therefore \quad x = 3.29 \Rightarrow$ 3.3 m.

answer (e)

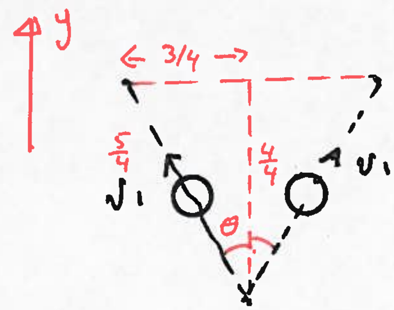
7) Little Willie, feeling real cool,
Thought he would play a little pool.
Alas, his luck was very cruel;
He lost the game, felt such a fool!

A.A.

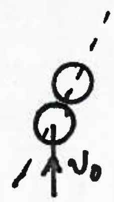


One of Willie's worst shots is shown below. He sends his white cue ball with velocity v so that it strikes a coloured ball at rest. To his dismay, both balls end up in different pockets, as shown! Using the given geometry, calculate the fraction of kinetic energy lost in the collision. Treat the balls as identical point masses and neglect any friction or rotational motion.

(a) 0 **(b)** 1/7 **(c)** 1/5 **(d)** 7/32 **(e)** 4/9



Conservation of momentum in the "x" direction shows that both balls move with the same speed v_1 at equal angles "θ" to the "y" direction.



$$(\Sigma p_i)_y = (\Sigma p_f)_y$$

$$m v_0 = 2 m v_1 \cos\theta$$

Since this is a 3, 4, 5 triangle $\cos\theta = 4/5$

$$\therefore \; v_0 = 2 v_1 \, 4/5 \Rightarrow v_1 = \frac{5}{8} v_0$$

now
$$\frac{\Delta KE}{(KE)_i} = \frac{(KE)_f - (KE)_i}{(KE)_i} = \frac{2\left(\frac{1}{2} m v_1^2\right) - \frac{1}{2} m v_0^2}{\frac{1}{2} m v_0^2}$$

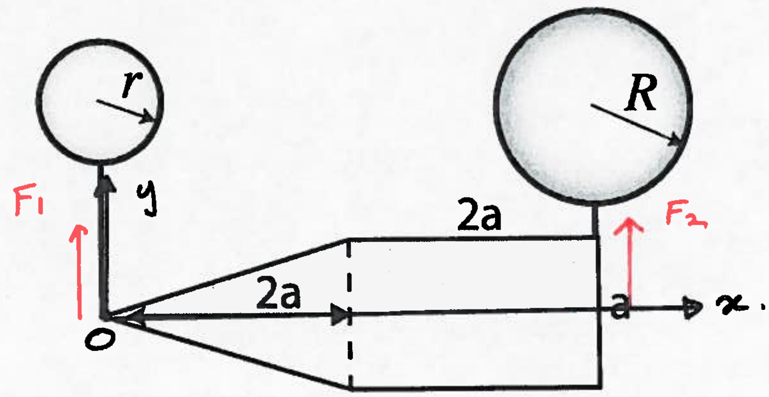
$$= \frac{2v_1^2 - v_0^2}{v_0^2} = \frac{\left(\frac{50}{64} - 1\right) v_0^2}{v_0^2} = -\frac{14}{64} = -7/32$$

The neg sign indicates a loss

$\therefore$ answer is d

8) In what appeared to the untrained eye as a meteor strike, an alien craft burned up in the Russian sky and landed in lake Chebarkul. A Russian salvage team, intent on extracting the alien DNA, models the escape pod as a flat plate of uniform thickness and composition (the aliens were very thin), as shown. To raise the pod to the surface, they plan to attach two spherical air-filled balloons of radius *r* and *R*, as shown. Note: the buoyant force is proportional to the weight of the water the balloons displace; also, the centre of mass of an isosceles triangle is located 1/3 of the way from the base to the apex. Determine the *ratio R/r* required, so that the pod stays level as it is raised.

(a) 1.08
(b) 1.16
(c) 1.25
(d) 1.57
(e) 1.75



The Buoyant forces

$F_1 = \frac{4}{3}\pi r^3 \rho$

$F_2 = 4/3\, \pi R^3 \rho$

We need to find the Center of mass about O for the "model"

We assume mass is proportional to area.

Triangular portion $m_1 = \frac{1}{2}(a)(2a)$

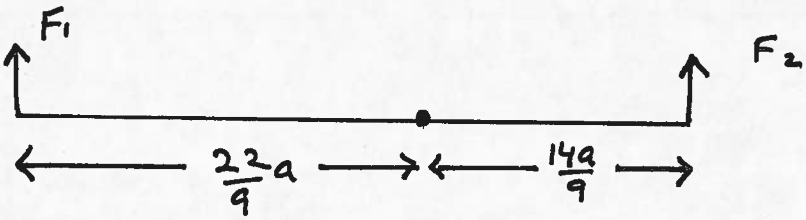
Rectangular portion $m_2 = (2a)(a)$

$$\vec{x}_{cm} = \frac{\sum m_c \cdot \bar{x}_c}{\sum m_c} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} \longrightarrow +ve.$$

$$= \frac{\frac{2}{3}(2a)\,\frac{1}{2}(2a^2) + 3a(2a^2)}{1/2\,(2a^2) + 2a^2} = \frac{\frac{4}{3}a + 6a}{3}$$

$$\vec{x}_{cm} = \frac{22}{9}a$$

now this system breaks down to the following.



now using the "Lever" method or the mechanical advantage method.

$$F_1\left(\frac{22}{9}a\right) = F_2\left(\frac{14}{9}\right)a.$$

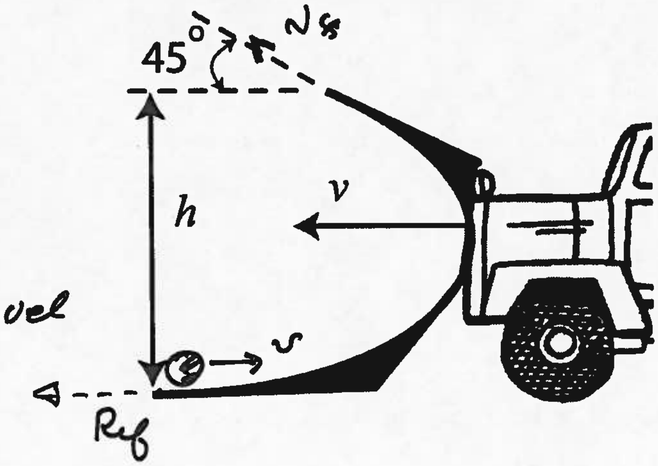
$$\therefore \frac{F_1}{F_2} = \frac{7}{11} = \frac{4/3\,\pi\,\rho\,r^3}{4/3\,\pi\,\rho\,R^3} \Longrightarrow \left(\frac{R}{r}\right)^3 = \frac{11}{7}$$

$$\therefore \frac{R}{r} = \left(\frac{11}{7}\right)^{1/3} = \underline{\underline{1.16}}$$

answer (b)

9) Power Plower Pete has designed a snow plow that will throw snow a great distance in the forward direction from the front of his monster truck, as shown. As the plow moves forward with speed v, assume that each chunk of snow it hits moves along it without friction or rotation. From the point of view of someone riding along with the truck, a chunk of snow enters the bottom of the plow with some kinetic energy, and leaves the top with some gravitational potential energy. If the *ratio* of kinetic to potential energies is 2:1, determine the speed of the snow as it leaves the top of the shovel *from the point of view of someone standing on the road.*

(a) $0.500\ v$
(b) $0.707\ v$
(c) $1.12\ v$
(d) $1.50\ v$
(e) $1.58\ v$



In the frame of the Shovel

$E_i = E_f.$

$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + mgh$

$\therefore v_f = \sqrt{v^2 - 2gh}$ — ①

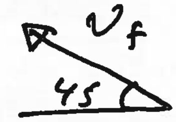
with reference to the Road.

$$\frac{(\text{Potential Energy})_f}{(\text{Kinetic Energy})_i} = \frac{mgh}{\frac{1}{2}mv^2} = \frac{1}{2} = \frac{2gh}{v^2}.$$

$\therefore \frac{2gh}{v^2} = \frac{1}{2}$ — ②

sub into ① $\quad v_f = \sqrt{v^2 - \frac{v^2}{2}} = \frac{1}{\sqrt{2}}v$

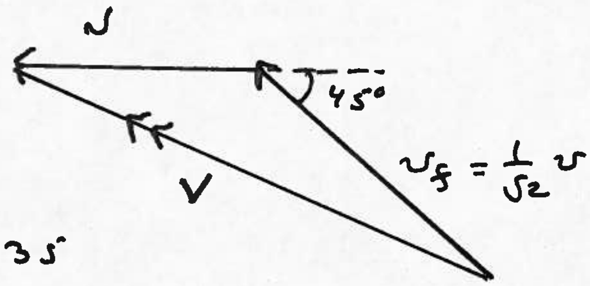
if $\bar{v}_{S,T}$ = Velocity of the snow relative to truck



$\bar{v}_{T,E}$ = " " " Truck relative to Earth $\xleftarrow{\ v\ }$

$\vec{V}_{S,E}$ = velocity of snow relative to Earth.

$$\vec{V}_{S,E} = \vec{V}_{S,T} + \vec{V}_{T,E}.$$



$$\therefore V^2 = v^2 + v_f^2 - 2 v v_f \cos 135$$

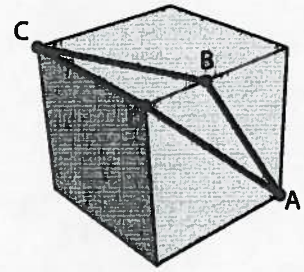
$$= v^2 + \frac{v^2}{2} - 2v\frac{v}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right)$$

$$= v^2 + \frac{v^2}{2} + v^2 = \frac{5}{2}v^2$$

$$\therefore V = \sqrt{\frac{5}{2}}\, v = \underline{\underline{1.58 v}}$$

answer ⓔ

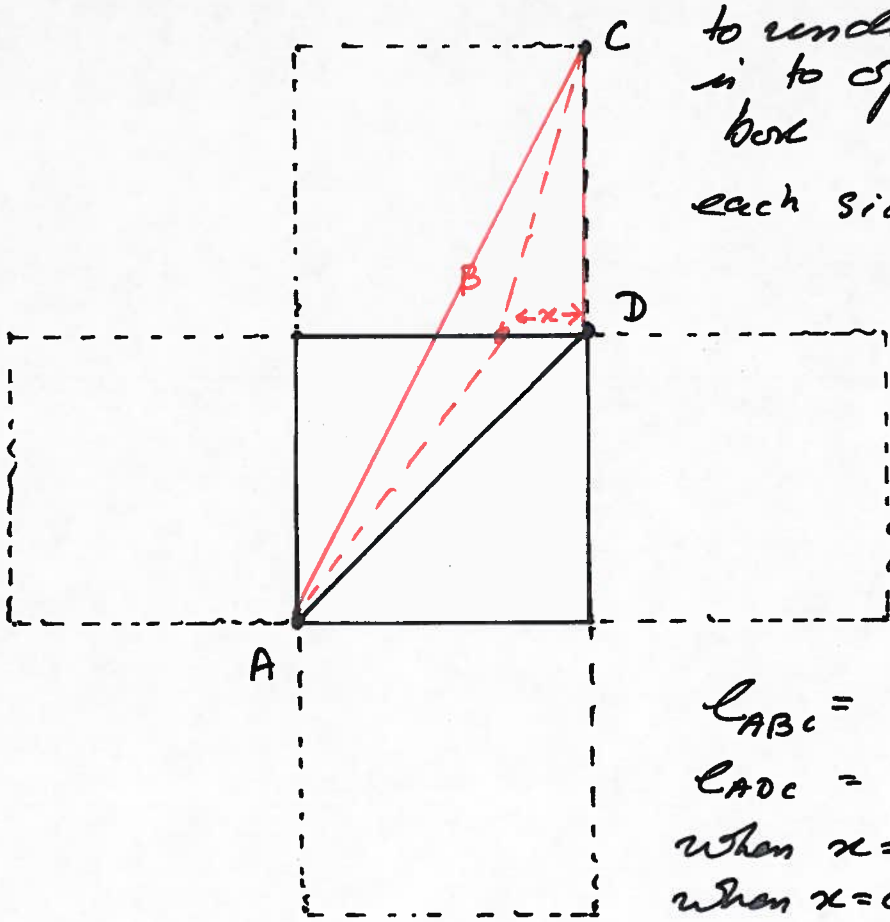
10) Two Trekkies in a role-playing fantasy are planning a fictional assault on a Borg ship in order to rescue their beloved Captain Picard. They imagine starting out at a corner of the cubic ship A, and want to crawl along the surface to get to the opposite corner C, and access an open hatch. One Trekkie will move along the trajectory ABC, where point B is anywhere along the edge of the cube. The second Trekkie will move along the trajectory ADC. Which Trekkie will reach his destination first, if their speeds are equal and they start simultaneously from point A?



(a) ABC
(b) ADC
(c) They reach simultaneously.
(d) The exact position of B is required.
(e) One cannot predict.

The easiest way to understand this is to open up the box

each side is of length "a"



l_{ABC} = length of ABC
l_{ADC} = " " ADC
when $x = 0$ or $x = a$
when $x = 0, x = a$
$l_{ABC} = l_{ADC}$

$l_{ADC} = \sqrt{a^2+a^2} + a = \sqrt{2}a + a = 2.41a.$

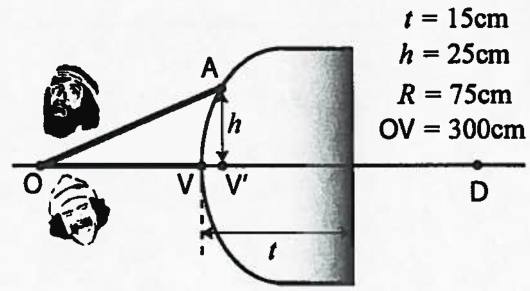
The minimum l_{ABC} is when B = is such that $x = a/2$.

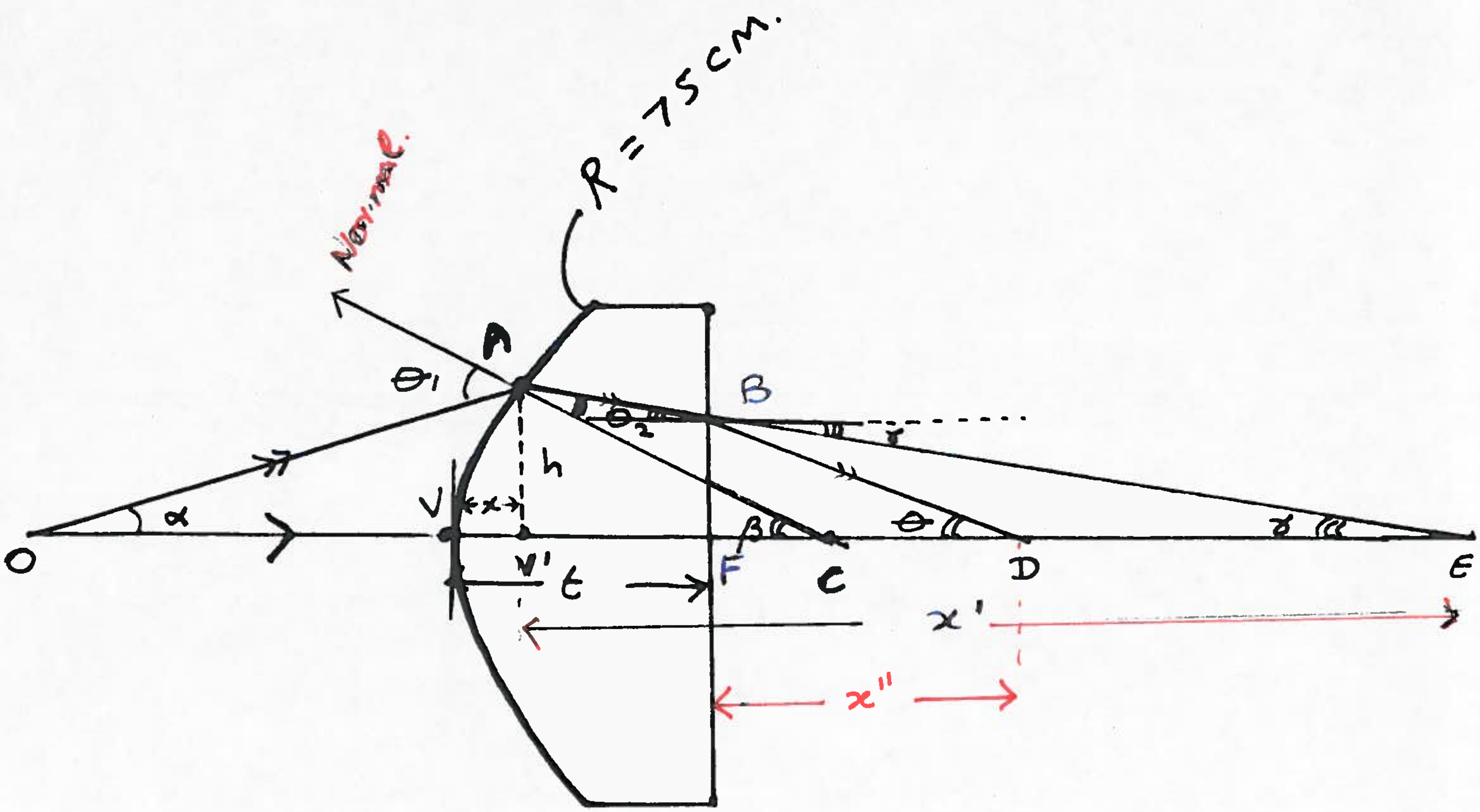
Then $l_{ABC} = 2\sqrt{a^2 + \frac{a^2}{4}} = \sqrt{5}\,a = 2.3a.$

$\therefore\ 2.3a \leq l_{ABC} \leq 2.41a = l_{ADC}.$ $\quad\therefore$ in general $l_{ABC} < l_{ADC}$

11) Cheech and Chong have just heard that red and green light refract at different angles. This insight made them curious, and they decided to test it with laser pointers. Using the spherical glass base of a hookah pipe of radius R, they perform their experiment in a smoky room. Cheech points his red laser at point "V" while Chong points his green laser at point "A". To their amazement, they realize the two beams meet at a point "D". How far was this point from their position at "O"? Use the index of refraction for air and glass as n_a=1, n_g=1.5, and answer in cm.

(a) 270 **(b)** 320 **(c)** 475 **(d)** 570 **(e)** 593





The Ray diagram is as shown. The Image is created at D. "C" is the centre of the curved surface

∴ AC is a Normal to the curved surface.

$OV = 300\text{cm} = \text{object distance} = p$
$R = 75 \text{ cm} = AC$
$h = 25\text{cm}$

$\alpha = \tan^{-1}\left(\frac{25}{304.29}\right) = 4.7^\circ$

$\beta = \sin^{-1}\left(\frac{25}{75}\right) = 19.47^\circ$ (from ACV' triangle)

$V'C = \sqrt{75^2 - 25^2} = 70.7 \text{ cm}$

$\therefore VV' = 4.29 \text{ cm}.$

$\theta_1 = (\alpha + \beta) = 24.2^\circ$

using snell's Law at A.

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow 1 \sin(24.2) = 1.5 \sin\theta_2.$

$\therefore \theta_2 = 15.8^\circ$

now $\beta - \theta_2 = \gamma = 3.67^\circ$

using $AV'E$ Triangle we see $\tan\gamma = \frac{25}{x'}$

$\therefore x' = 389.76 \text{ cm}.$ let $y = BF$

Hence using the Triangles $AV'E$ and BFE we see that $\tan\gamma = \frac{h}{x'} = \frac{y}{x' - t}$

$\frac{25}{389.76} = \frac{y}{389.76 - 15} \Rightarrow y = 24.04$

$\therefore \tan\theta = y/x''$ use Snell's Law at B

$$\left.\begin{aligned} n_2 \sin\gamma &= n \sin\theta \\ 1.5 \sin(3.67) &= \sin\theta \end{aligned}\right\} \therefore \theta = 5.38^\circ \simeq 5.4$$

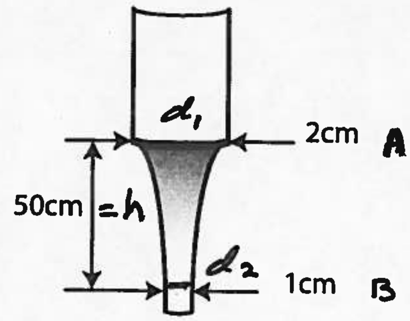
Hence $\tan\theta = y/x'' \Rightarrow x'' = 254.4 \text{ cm}.$

Hence the distance from $O = 15 + 300 + 254.4 \simeq 569.3$
$\simeq 570 \text{ cm}.$

answer (d)

12) Bob McKenzie is pouring a "strange brew" for his brother Doug at Kitchener-Waterloo's Oktoberfest, when the pair decides to calculate the flow rate from the keg. The brew flows out from a downward-facing tap 2cm in diameter, and accelerates as it free falls, with the stream diameter reducing on the way down from the tap. 50cm down, the stream is 1cm in diameter. What is the flow rate in cm^3 per second? You may assume the incompressible brew always flows vertically, and that surface tension keeps the stream circular, but does not influence the vertical motion.

(a) 140 **(b)** 183 **(c)** 212 **(d)** 230 **(e)** 254



we could say

$$v_B^2 = v_A^2 + 2gh \quad (1) \text{ using The kinematics eqn.}$$

The flow rate at A and B are the same.

$$v_A d_1^2 = v_B d_2^2 . \quad (2)$$

substitute (2) into (1)

$$\left(v_A \frac{d_1^2}{d_2^2}\right)^2 = v_A^2 + 2gh$$

$$v_A^2 = \frac{2gh}{\left(\frac{d_1}{d_2}\right)^4 - 1}, \quad \text{flow rate} = v_A \frac{\pi}{4} d_1^2$$

$$= \frac{\frac{\pi}{4} d_1^2 \sqrt{2gh}}{\sqrt{(d_1/d_2)^4 - 1}}$$

$$= \underline{254 \text{ cm}^3/\text{sec}}$$

answer (e)