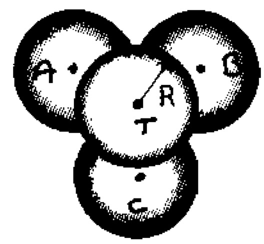
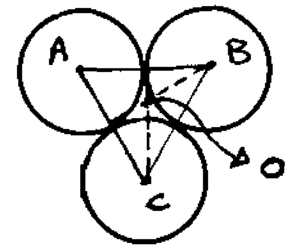
1) Laura Secord was making solid spherical cannon balls of radius R for General Brock during the war of 1812. She placed three of them on her smooth level floor. Each was in contact with both of the other two, and contact was maintained by a thin layer of chocolate syrup. She then placed a fourth ball into the "pocket" formed by the first three, with three more dabs of chocolate at the contact points. Calculate the minimum vertical clearance under the bottom shelf in her closet to allow her to slide the pile along the floor and out of sight of the American invaders.

(a) 3.19R
(b) 3.33R
(c) 3.63R
(d) 3.91R
(e) None of the above.





A, B and C are centres of the cannon balls placed on the floor.

$\therefore AB = BC = AC = 2R$

In the same way $AT = BT = CT = 2R$

All angles in the triangle $ABC = 60°$

Hence $\hat{ACO} = 30°$

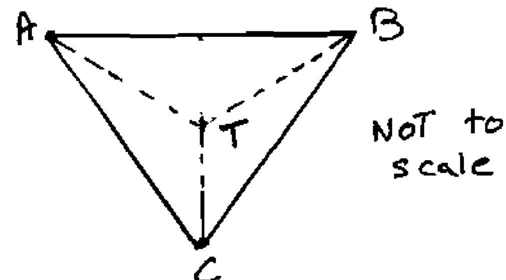
$\therefore OC = \frac{2}{3}\left[2R\cos(30°)\right] = OB$

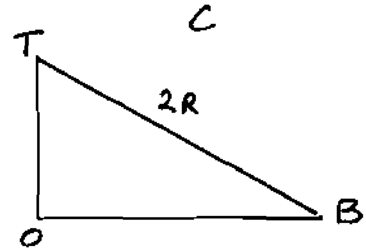
$\therefore OT = \left[(TB)^2 - (OB)^2\right]^{1/2}$

$= \left[(2R)^2 - \left[\frac{2}{3}(2R)\cos 30°\right]^2\right]^{1/2}$

$= \left[4R^2 - \frac{16}{9}R^2\cos^2 30\right]^{1/2}$

$OT = 1.633R.$



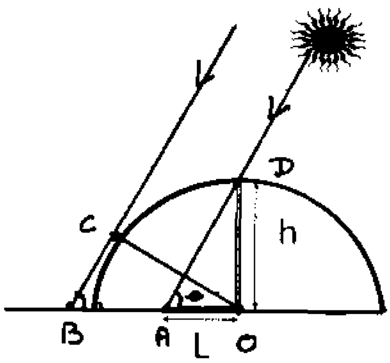


$\therefore$ From the floor to the top of the "Top" Cannon it will be $2R + 1.633R = \underline{3.633R}$

$\therefore$ answer is c

2) A rigid metal pipe packed with recycled robocall records was sent to a remote prairie location, where it was mounted vertically. The tall pipe (h=5m) was illuminated by the sun and cast a shadow (L=3m) on the level ground. Suddenly, a gust of wind causes the pipe to pivot in the direction of the cast shadow with no skidding at the base. The length of the shadow increases up to some point, and then starts to decrease. Calculate the maximum length of the shadow during the fall to the ground. Answer in m.

(a) 3.54
(b) 4.01
(c) 4.68
(d) 5.23
(e) 5.83



The maximum length of the shadow will be OB
BC is tangent to the circle.

$OC = OD = h$ $\quad OC \perp BC$ since the radius is perpendicular to the tangent. If $D\hat{A}O = C\hat{B}O = \theta$

$$\tan\theta = \frac{OC}{BC} = \frac{h}{BC} \qquad \tan\theta = \frac{OD}{OA} = \frac{h}{OA} = \frac{h}{3}$$

OA is given to be = 3 m

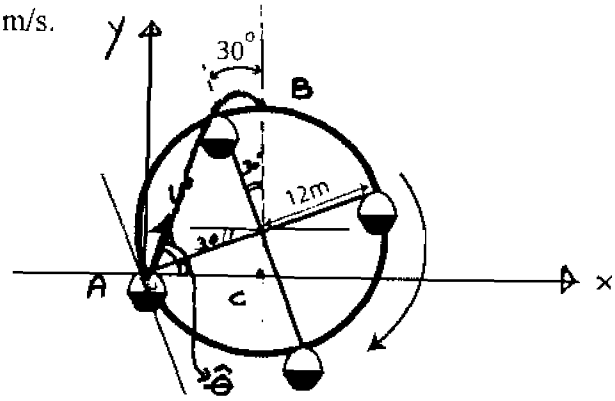
$\therefore BC = 3$ m.

$$\therefore OB = \left[(OC)^2 + (BC)^2\right]^{1/2}$$
$$= \left[5^2 + 3^2\right]^{1/2}$$
$$= \sqrt{34} \text{ m}$$
$$\therefore OB = \underline{5.83 \text{ m}}$$

answer e

3) Dion Faneuf, captain of the Toronto Maple Laughs, has a new job with Cirque du Soleil. Wearing his blue and white clown suit, he rides on the level seat of a small open bucket at the rim of a 12m radius vertical ferris wheel, rotating steadily at 5 revolutions per minute. While passing through the position as shown, he throws a juggling ball which leaves the bucket with a relative velocity of V'. Calculate the magnitude of V' so that he can catch the ball just as it returns to his bucket at the top of the ride. Answers in m/s.

(a) 3.1
(b) 6.3
(c) 15.6
(d) 19.5
(e) 24.2



$5\ Rev/min = 0.5236\ rad/sec$
$= 30°/sec = \omega$

ω is the angular speed.

The time taken for Dion's bucket to reach the top = 4 sec.

using our Reference frame as shown

The Vertical displacement = $BC = 12 + 12\sin 30 = 18\ m$

The horizontal displacement = $AC = 12\cos 30 = 10.4\ m$.

let V_0 be the initial velocity relative to Earth = $v_{B,E}$

$\therefore$ for the Vertical displacement of the ball

$\uparrow\ \bar{y} - \bar{y}_0 = V_{0y} t + \frac{1}{2} a_y t^2 \Rightarrow 18 = V_0 \sin\theta\, t - \frac{1}{2}(9.8) t^2$

$18 = V_0 \sin\theta (4) - 4.9(16)$

$\therefore\ V_0 \sin\theta = 24.1$ —— ①

For horizontal motion $\longrightarrow$ $\bar{x} - \bar{x}_0 = \bar{V}_{0x} t$

$10.4 = V_0 \cos\theta (4)$

$\therefore\ V_0 \cos\theta = 10.4$ —— ②

From ① and ② $\Rightarrow$ $V_0 = 24.24\ m/s$

$\theta = 83.84°$

now The Velocity of The Ferris wheel can be found as $v_t = R\omega \Rightarrow (12)(0.5236) = 6.28\ m/s$ Tangent to the circular path.

P.T.O

∴ $\vec{V}_{F,E}$ = Velocity of Ferris wheel relative to Earth

$= 6.28\ m/s$ where $\hat{\alpha} = 30^\circ$

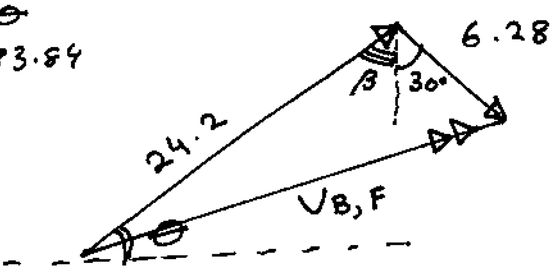
$\vec{V}_{B,E} = \vec{v}_o = 24.24\ m/s$ $\hat{\theta} = 83.84$

∴ $\vec{V}_{B,F} = V'$ = Velocity of ball relative to Ferris wheel

$$\vec{V}_{B,F} = \vec{V}_{B,E} + \vec{V}_{E,F}$$

∴ adding these Vectors graphically

$$\beta = 90 - \theta = 90 - 83.84 = 6.16^\circ$$



$$\therefore V_{BF} = V_1 = \left[(24.2)^2 + (6.28)^2 - 2(24.2)(6.28)\cos(36.16)\right]^{1/2}$$

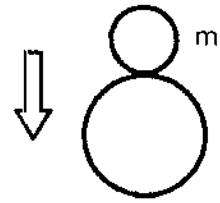
$$= 19.48 \approx 19.5\ m/s$$

answer d.

3 continued.

4) Little Willie from his hand,
Dropped two balls and watched them land.
The lighter one then bounced quite high.
He scratched his head and wondered why.

A.A.



Two very small balls are held as shown and then dropped vertically from a height h. with the heavier ball, mass M, directly below the lighter one, mass m. The balls remain in contact until the first collision with the ground. Assume that the balls may be treated as point masses, all collisions are elastic, and air resistance is negligible. Find the ratio h_1/h_2, where h_1 is the maximum height reached by the lighter ball and h_2 is that of the heavier ball after the collision, given that M/m=7.0.

(a) 7.0
(b) 10.0
(c) 14.0
(d) 25.0
(e) 35.0

Let the common speed of the two balls just before hitting the ground be V

Then conservation of energy: $\frac{1}{2}(M+m)V^2 = (M+m)gh$
$V^2 = 2gh$ — ①

The heavier ball will rebound upward with speed V and instantaneously collide with the lighter ball which has a downward sped V. After the collision, let the lighter ball have speed v_1 and heavier ball v_2 both upwards.

↑ up as positive, conservation of momentum $MV - mV = Mv_2 + mv_1$

Since the collision is elastic, we can use velocity of approach equals velocity of separation

$$V + V = v_1 - v_2 \quad — ②$$

We eliminate v_2: $MV - mV = M(v_1 - 2V) + mv_1$

$$\therefore \; v_1 = \frac{(3M - m)}{M+m} V$$

Since $M = 7m$ $\quad v_1 = \frac{20}{8} V = \frac{5V}{2}$ and from ② $v_2 = v_1 - 2V$
$= V/2$

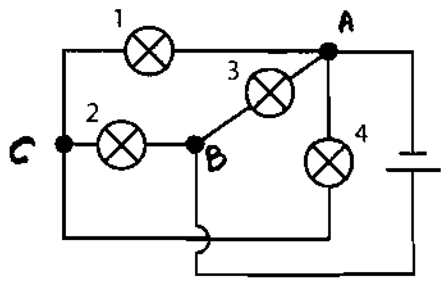
$\therefore \frac{v_1}{v_2} = 5$ now using ① we find that

$$\frac{h_1}{h_2} = \left(\frac{v_1}{v_2}\right)^2 = 25$$

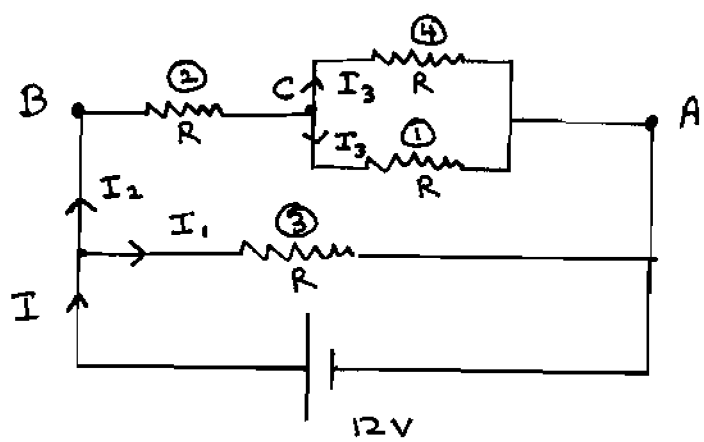
answer d

5) While piloting his submarine through a bottomless lake in northern Manitoba, Josiah Flintabbatey Flonatin passes through a hole lined with gold into a strange underground world. In order to illuminate his voyage, he attaches a circuit rigged with four lightbulbs to the front of his craft, pictured below. If the circuit uses identical bulbs connected to a 12 V battery as shown, list the bulbs in order of increasing brightness.

(a) (1&2), 3, 4
(b) 1, (2&3), 4
(c) (1&4), 2, 3
(d) (1&4), (2&3)
(e) 1, 2, 3, 4



We can simplify the circuit to something like this



The total Resistance of ②, ④ and ①

$\frac{3}{2} R\ \Omega$.

$\therefore I_2 \left(\frac{3}{2} R\right) = I_1 R = 12V$

$\therefore I_1 > I_2$

we also see that $I_2 = 2 I_3 \quad \therefore I_3 = I_2/2$

$\therefore I_1 > I_2 > I_3$

① & ④ will have the same brightness. The brightness will depend on the current.

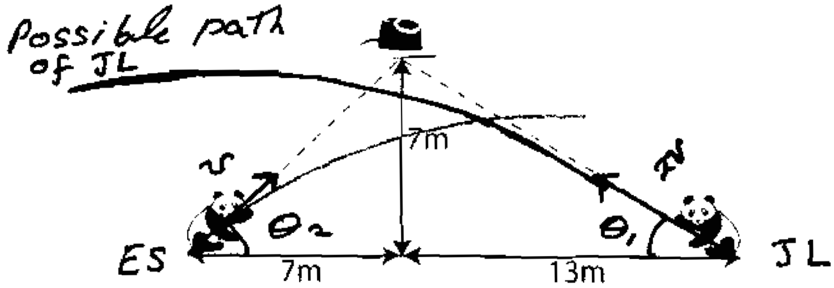
$\therefore$ I_1 is the smallest while I_3 is the largest

$\therefore$ [① and ④] then ② and then ③

$\therefore$ answer C

6) Canada's new giant pandas, Er Shun and Ji Li, break out of their enclosure at the Calgary Zoo, and wander into the Stampede parade. Standing 20m apart from each other, they spy a spiffy cowboy hat 7m above the ground, being worn by a man on stilts. Er Shun is 7m away from the man, and Ji Li is 13m away, when they start to throw rocks to dislodge the hat. Being pandas, they throw their rocks aiming straight for the hat, not accounting for the acceleration due to gravity. Ji Li throws her stone at twice the speed of Er Shun, whose stone strikes the ground 3 seconds after it leaves his paw. By what vertical distance will the stones miss each other if thrown at the same instant? Answer in m.

(a) 1.98
(b) 2.42
(c) 3.03
(d) 3.61
(e) 4.10



$V_{JL} = 2v \quad V_{ES} = v$

$\tan\theta_1 = \frac{7}{13} \Rightarrow \theta_1 = 28.3°$

$\tan\theta_2 = \frac{7}{7} = 1 \Rightarrow \theta_2 = 45°$

Er Shun (ES) stays in flight for 3 sec.

Hence if we look at the vertical displacement of ES

$\uparrow\ y - y_0 = V_{oy}t + \frac{1}{2}a_y t^2 \Rightarrow 0 = v\sin 45\, t - \frac{1}{2}(9.8)t^2$

since $t = 3$ sec.

$v = 20.79$ m/s

at sometime "t" they would be directly above one anoth

The horizontal displacement at this time

for JL $\leftarrow$ $\Rightarrow$ $x - x_0 = V_{ox}t + \frac{1}{2}a_x t^2, \quad a_x = 0$

$x = 41.6\cos(28.3)\, t$ —— ①

for ES $\rightarrow$ $\Rightarrow$ $20 - x = 20.79\cos 45\, t$ —— ②

①/② $\Rightarrow \frac{x}{20-x} = \frac{41.6\cos 28.3}{20.7\cos 45} \Rightarrow x = 14.29$ m

$\therefore t = 0.39$ sec.

Hence for vertical displacements using $y_1 - y_0 = V_{oy}t + \frac{1}{2}a_y t^2 \uparrow$

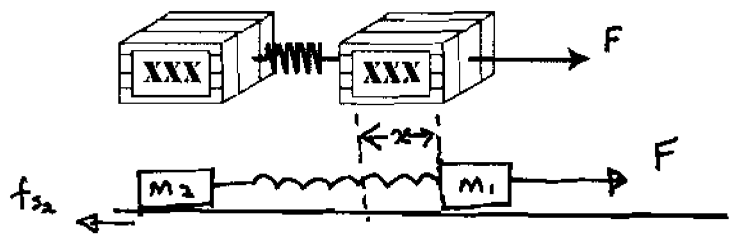
for ES $\Rightarrow$ $y_1 = (20.7)\sin 45(0.39) - 4.9(0.39)^2 = 4.96$

for JL $\Rightarrow$ $y_2 = (41.6)\sin(28.3)(0.39) - 4.9(0.39)^2 = 6.95$

$\therefore y_2 - y_1 = 1.98$ m // answer a

7) In order to supply the USA with whisky during prohibition, Joseph Seagram devised a way to drag crates of "moonshine" through tunnels across the border. At the tunnel entrance in Waterloo, two crates, each of mass m, are connected by a non-stretched spring (with spring constant k) and lie flat on the horizontal tunnel floor. If the coefficients of static and kinetic friction between crate and floor have the same value μ, with what *minimum constant* force must Joseph pull on one of the crates, in order to move the second one? Answer in μmg.

(a) 1
(b) 1.5
(c) 2
(d) 2.5
(e) 3



If m_2 is on the verge of moving

Using $\sum \vec{F}_x = ma_x \rightarrow \Rightarrow kx - f_{s_2} = 0$

$\therefore \; kx - \mu mg = 0$ — ①

now the work done on m_1, by the force and spring using the work energy theorem.

$$Fx - (f_{s_1})x - \frac{1}{2}kx^2 = \Delta K.E = \frac{1}{2}m_1(v_f^2 - v_i^2)$$

$v_i = 0$ since it starts at rest. $f_{s_2} = (\mu mg)x$

$$\therefore \; Fx - \mu mgx - \frac{1}{2}kx^2 = \frac{1}{2}mv_f^2$$

$$\therefore \; F = \frac{\frac{1}{2}mv_f^2 + \frac{1}{2}kx^2 + \mu mgx}{x}$$

If we need to minimize F, the only variable we could minimize is v_f. let $v_f \rightarrow 0$

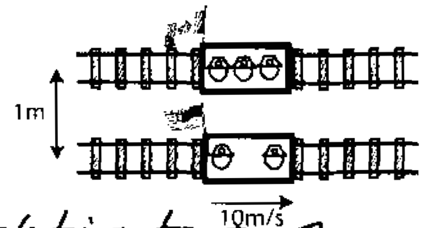
$$\therefore \; F_{min} = \frac{1}{2}kx + \mu mg$$

from ① $kx = \mu mg$. $\therefore$ $F_{min} = 1.5\,\mu mg$

answer b

8) Two groups of miners, one Canadian and one American, are coasting to a gold field in the Yukon. They are in small massless carts, on level, frictionless tracks, side-by-side as shown. The Canadian cart (3 people of 100kg each) got a head start on the American cart (2 people of 100kg each), but the Americans got a faster launch of 10m/s. As the Americans catch up, a Canadian leaps from his cart into the American cart. To achieve this, he jumps with a forward velocity component (along the direction of the track) of 2m/s relative to the Canadian cart. But this component is also 1m/s in the same direction relative to the American cart. What are the final velocities of the two carts right after the defector has come to rest in the American cart? Answer in m/s.

(a) 8.00 and 10.33
(b) 9.00 and 10.33
(c) 8.00 and 11.00
(d) 9.00 and 11.00
(e) 10.33 and 11.00



Let $V_{P,E}$ = Velocity of person relative to Earth.
$V_{P,C}$ = " " " " to The Canadian Cart
$V_{P,A}$ = " " " " " " american cart

$\therefore \vec{V_{P,E}} = (\vec{V_{P,C}})_{after} + (\vec{V_{C,E}})_{after}$ —— ①

after ⇒ an instant after the jump off The cart

$\vec{V_{P,E}} = (\vec{V_{P,A}})_{before} + (\vec{V_{A,E}})_{before}$ —— ②

before ⇒ an instant before the landing on the cart

$\vec{V_{P,E}} = 1 \rightarrow + 10 \rightarrow = 11\ m/s \rightarrow$

now if we sub This into ① $\Rightarrow (V_{CE})_{after} = 9\ m/s \rightarrow$

now using Conservation of momentum for The american cart $\sum \vec{p_i} = \sum \vec{p_f}$

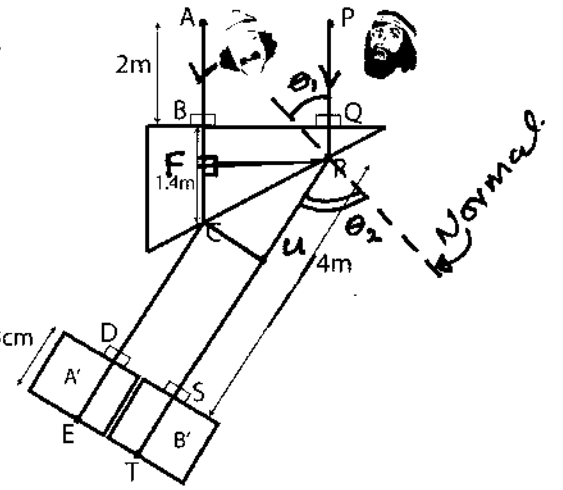
$\rightarrow m_p v_{P,E} + m_A v_{A,E} = (m_p + m_A)(v_{A,E})_{after}$

$(100)(11) + (200)(10) = (300)(v_{A,E})_{after}$

$\therefore (v_{A,E})_{after} = \frac{31}{3} = 10.3\ m/s \rightarrow$

$\therefore$ answer b

9) Cheech and Chong shine identical laser pointers through a triangular piece of glass with index of refraction n(g)=1.5, 2m away. Chong then places two transparent blocks of different indices of refraction n(A')=1.4 and n(B')=1.7, so that the rays incident on them are perpendicular to the surfaces. He introduces the term "Optical Path Length" to Cheech as the geometrical length multiplied by the index of refraction in that medium. He asks Cheech to compare his Optical Path Length of ABCDE to Chong's Optical Path Length of PQRST. What is the difference in the two? Answer in cm.



(a) 0
(b) 0.9
(c) 1.4
(d) 1.7
(e) 9.3

OPL = optical path length.
we see that
$(OPL)_{AF} = (OPL)\ PR$

in the same way
$(OPL)_{CD} = (OPL)_{us}$

We draw a Normal to the Surface RC at R

$\therefore n_g \sin\theta_1 = n_a \sin\theta_2$ $\qquad n_a$ = I.R. of air

but $F\hat{R}C = \theta_1$ and $R\hat{C}u = \theta_2$

$\therefore n_g\left(\frac{FC}{RC}\right) = n_a\left(\frac{Ru}{RC}\right) \Rightarrow n_g(FC) = n_a(Ru)$

$\therefore (OPL)_{FC} = (OPL)_{Ru} \quad \therefore (OPL)_{ABCD} = (OPL)_{PQRS}$

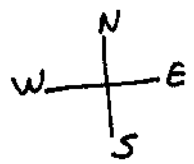
$\therefore$ The difference in optical path length's would be

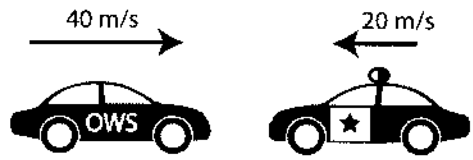
$[n(B')(ST) - n(A')(ED)] = (1.7 - 1.4)(3) = 0.9$ cm.

answer b

10) Two Occupy Wall Street protesters are going for a joy-ride in a BMW stolen from a corporate executive. Traveling North on the highway at 40m/s, they meet a police car traveling South at 20m/s. The police car can both decelerate and accelerate at $5 m/s^2$, but has to slow to 4m/s before making a "cop-turn" which involves an instantaneous change in direction (but not speed). How long, measured from the time they first met, will it take before the police car can catch up with the protesters and give them a thorough pepper spray-down? Answer in s.

(a) 6.10
(b) 7.00
(c) 9.20
(d) 18.1
(e) 21.2





Our time measurements begin the instant they crossed.

Police car. using $\bar{v} = \bar{v}_0 + at$

$4 = 20 - 5t$ ↓

$t = 3.2$ sec.

BMW

↑ $40 \times 3.2 = 128$ m

distance travelled.

↓ $y = v_0 t + \frac{1}{2} a t^2$

$= (20)(3.2) - \frac{1}{2}(5)(3.2)^2$

$= 38.4$ m

∴ When the Policeman switches directions they are $128 \text{ m} + 38.4 \text{ m} = 166.4$ m apart.

Say it now takes some time "t" for them to meet. Then the distance the policeman has to travel will be

$(40t + 166.4)\,m = S = ut + \frac{1}{2}at^2$.

∴ $40t + 166.4 = 4t + \frac{1}{2} 5 t^2$

Solving this quadratic

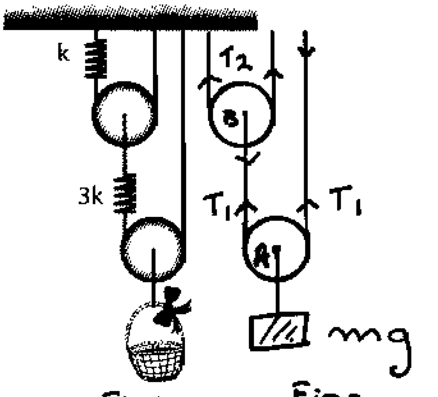
$t = 18.08$ sec.

∴ total time $= 18.08 + 3.2 \approx 21.2$ sec.

answer e

11) Hiking alone in the Canadian wilderness, Little Red Riding Hood fears that wolves will eat her basket of goodies. She rigs up a system of pulleys, ropes, and springs to hang her basket out of harm's way. The springs have spring constants k=100 N/m and $3k$=300 N/m. She ties her 8kg basket to the bottom pulley, then lowers it slowly with both hands until it is supported in static equilibrium. Find the distance by which the lower pulley descends. Everything except her basket is massless; the ropes are unstretchable. Answer in m.

(a) 0.114
(b) 0.146
(c) 0.163
(d) 0.500
(e) 1.167



In Fig. 2

for pulley A

$2T_1 = mg$

for pulley B, $2T_2 = T_1$

$\therefore \; T_1 = \frac{8g}{2} = 4g$

$T_2 = \frac{T_1}{2} = 2g$

now using Fig 1 $\quad T_1 = 3ky_1 = 4g$

$\therefore \; y_1 = \frac{4g}{3k}$

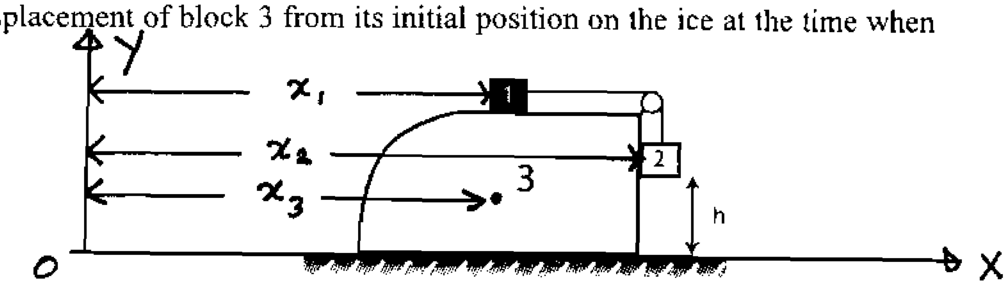
$y_2 = \frac{T_2}{k} = \frac{2g}{k}$. $\therefore$ pulley B would descend $y_2/2$

$\therefore$ The lower pulley would descend $\frac{y_1 + y_2/2}{2} = Y$

$\therefore Y = \frac{\frac{4g}{3k} + g/k}{2} = \frac{7g}{6k} = \frac{7(9.8)}{600} = 0.114$ m.

12) Hit by rising prices, Gary Bettman makes a fuel-free Zamboni to save the NHL some money. The Zamboni is composed of a system of blocks as shown, initially at rest, such that the bottom of block 2 is held at a height *h* from the frictionless ice, which supports block 3. The coefficients of static and kinetic friction between blocks 1 and 3 are equal, $\mu_s=\mu_k=\mu$. After block 2 is released from its position, it starts sliding down along the right side of block 3. Blocks 1 and 2 are connected by a massless rope, and block 2 is constrained to stay in (frictionless) contact with block 3. What is the displacement of block 3 from its initial position on the ice at the time when block 2 hits the ice?

(a) $h \times m_1/(m_1+m_2+m_3)$
(b) $\mu \times h \times (m_1+m_2)/m_3$
(c) $h \times m_2/(m_1+m_3)$
(d) $h \times m_2/(m_1 \times \mu)$
(e) $h \times (m_1+m_2)/m_3$



There are no net external forces acting on the system in the X direction.

∴ Center of mass of the system relative to "O" is conserved.

$$\therefore \sum (X_{cm})_i = \sum (X_{cm})_f.$$

$$\left(\sum X_{cm}\right)_i = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

When mass #2 slides down "h" units, mass m_1 will slide "h" units to the right.

∴ The system will move "D" units to the left.

$$\therefore \left(\sum X_{cm}\right)_f = \frac{m_1(x_1 + h - D) + m_2(x_2 - D) + m_3(x_3 - D)}{m_1 + m_2 + m_3}$$

$$\therefore \sum (X_{cm})_i = \sum (X_{cm})_f$$

$$\cancel{m_1 x_1} + \cancel{m_2 x_2} + \cancel{m_3 x_3} = m_1 \cancel{x_1} + m_1 h - m_1 D + \cancel{m_2 x_2} - m_2 D + \cancel{m_3 x_3} - m_3 D$$

$$\therefore (m_1 + m_2 + m_3) D = m_1 h$$

$$\therefore D = \frac{m_1 h}{m_1 + m_2 + m_3} \qquad \text{answer a}$$