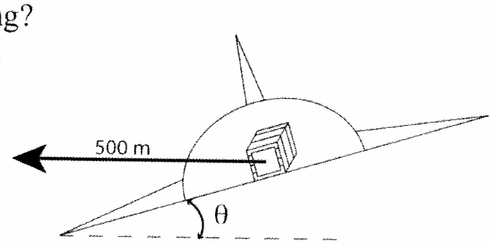
2011 - 43nd SIN Exam

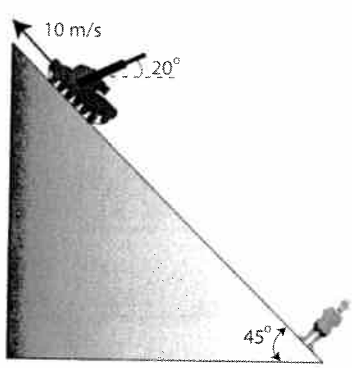
1) A large refrigerated cargo plane is delivering crates of ice cubes to the arctic as part of the conservative government's Economic Action Plan. The crates sit unsecured in the middle of the flat floor. Near the drop zone, the plane begins flying in a uniform horizontal circle at 150 m/s. If the radius of the circle is 500 m, and the plane's wings are tilted at an angle θ from the horizontal so that a force on the plane is provided by the lift perpendicular to the wing surface, what is the minimum coefficient of static friction required to prevent the crates from sliding?

(a) 0.000
(b) 0.110
(c) 0.217
(d) 0.323
(e) 0.434



2) Radical Albertan separatists are driving a tank at a constant speed of 10 m/s up a 45 degree mountain slope, in a vain attempt to invade British Columbia. Pursued by the Mounties, the tank fires a 10 kg watermelon, with muzzle velocity 25 m/s, back along its path. Ignore the distance from the muzzle to the slope. If the gun barrel is pointed 20 degrees above the horizontal as shown, how far is the melon from the tank when it finally strikes the slope below? Answer in m.

(a) 84.6
(b) 101
(c) 152
(d) 217
(e) 269

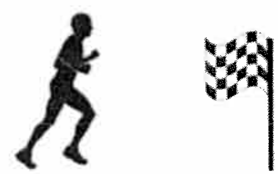


3) Little Willie in a sprint race,
Started off at a super pace;
Ran out of steam, as others passed.
Yes you've guessed it - he finished last!

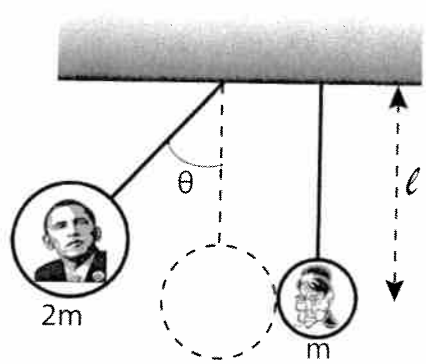
A.A.

Willie, in a 100 m race, initially accelerates uniformly from rest at 2.00 m/s^2 until reaching his top speed of 12.0 m/s. He maintains this speed, until he is 16.0 m from the finish line, but then fades and decelerates uniformly, crossing the line with a speed of only 8.00 m/s. What was Willie's total time for the race? Answer in seconds.

(a) 10.8
(b) 11.2
(c) 11.6
(d) 12.0
(e) 12.5

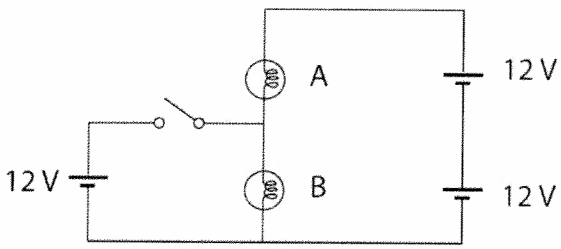


4) Barak Obama proposes a TV discussion with Sarah Palin. They design a special set, where each sits in a plastic sphere (so that their center of mass is at the center of the sphere), suspended at a distance ℓ from the ceiling. The separation at the ceiling keeps both ropes vertical when the spheres contact. Obama's sphere (twice the mass of Palin's, who is a known lightweight) is pulled to the left, and released from θ = 60 deg to the vertical as shown. What maximum angle from the vertical does Palin's sphere reach on her first swing, if the collision is completely elastic? Answer in degrees.



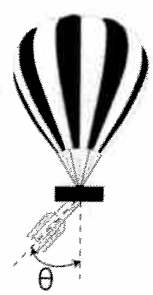
(a) 20.2
(b) 41.8
(c) 62.1
(d) 83.6
(e) Greater than 90 (she hits the ceiling)

5) The light bulbs in the circuit pictured below are identical. When the switch is closed:



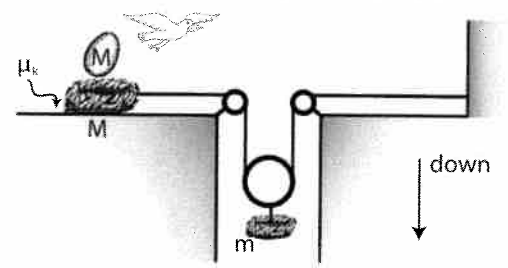
(a) both lights go out.
(b) one of the lights goes out.
(c) the intensity of light bulb A increases.
(d) the intensity of light bulb B increases.
(e) nothing changes.

6) Peter Mackay, learning from Fox News that the Russians are en-route to claim the North Pole, commandeers a hot air balloon to confront them. While cruising with the polar airflow at an altitude of 1 km with a constant velocity of 15 km/h, the temperature in the balloon suddenly drops, reducing its buoyant force by 10%. Fortunately, he has a jet system that can create a thrust, so that, if pointed directly down, it would exactly compensate for the lost buoyancy. In order to claim the Pole, Mackay needs to travel an additional horizontal distance of 2 km within 2 minutes, and a quick calculation shows that he won't make it without assistance. He sets the angle θ equal to 30 degrees and turns on the jet. What will his altitude be as he passes over the Pole? Neglect any air resistance.



(a) He would pass over the Pole too late.
(b) He will hit the ground too soon and be eaten by a polar bear.
(c) 556 m
(d) 471 m
(e) 293 m

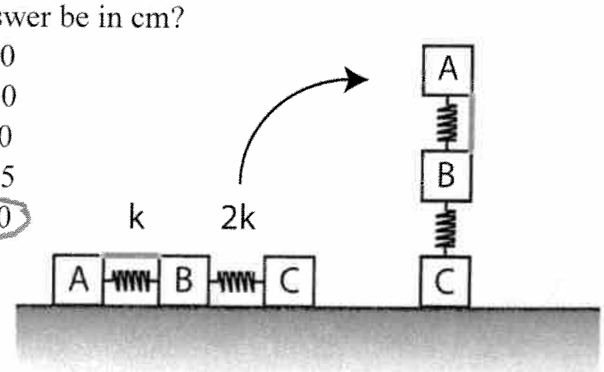
7) A bird watcher sets up an experiment with two bird nests, mass M and m, which are attached to a system of frictionless massless pulleys via a single rope, and released as shown. Before her second trial, a bird lays an egg into one nest, doubling its mass from M to 2M. If the sliding nest's acceleration decreases from g/2 to g/4 between the two trials, what is the coefficient of kinetic friction μ_k between the nest and the horizontal plane?



(a) 0.10
(b) 0.26
(c) 0.35
(d) 0.60
(e) 0.72

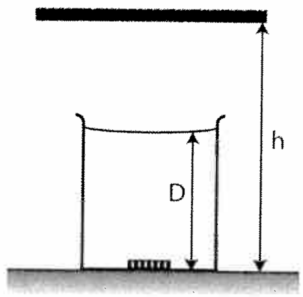
8) Michael Ignatieff and Jack Layton decide to see if Jim Flaherty can calculate a decent budget. First, they bring three blocks each of mass 0.5 kg and connect them in line to one another on a horizontal table using stiff springs. The springs, with constants k=1000 N/m and 2k, are at their equilibrium lengths. Next, blocks A and B are connected via a massless string which is tightened to a force of F=20 N. Finally the whole system is tipped upright onto block C as shown in the figure. They ask Flaherty what the total change in the distance between blocks A and C is in going from the original horizontal to the final vertical configuration. What should his answer be in cm?

(a) 0.50
(b) 1.50
(c) 2.00
(d) 2.25
(e) 2.50



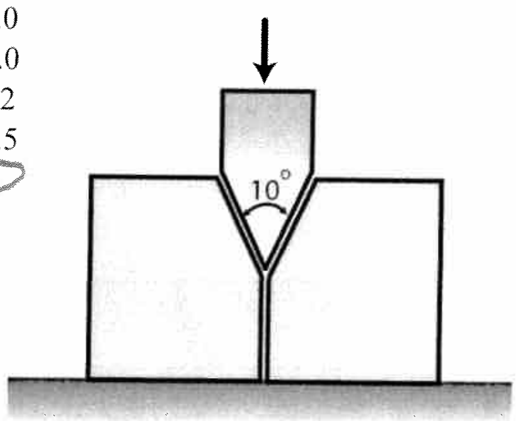
9) Cheech finds his favorite drinking glass and decides to place a coin at the bottom of it. He then places a flat mirror a distance h above the base and asks Chong to predict the position of the image of the coin. Chong accepts the challenge but now fills the glass to a height D with his favorite drink (index of refraction 1.33) and challenges Cheech to predict the distance from the base of the glass to the image of the coin in the mirror. Assuming the small-angle approximation, what would Cheech's correct answer be?

(a) h-D/2
(b) 2h-D/4
(c) h-D/4
(d) h-D
(e) 2h-D



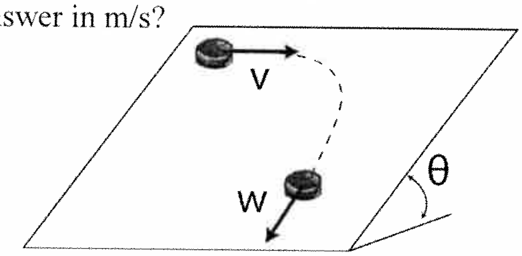
10) Romeo tries to impress Juliet by showing off what he created in shop class. He has machined two large blocks, each of mass 1500 kg, with a precise angle as shown in the diagram. He puts them on a very smooth surface, and on top places a vertical 10 degree symmetric wedge of mass 5 kg. He then asks Juliet to apply a gentle force on the wedge, so as to accelerate the two blocks sideways at 0.5 m/s^2. How much force (in Newtons) does Juliet apply? Assume that there is no friction anywhere.

(a) 45.0
(b) 62.0
(c) 75.2
(d) 82.5
(e) 111



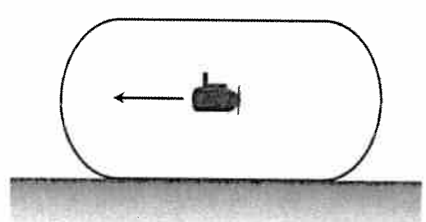
11) The Toronto Maple Laughs, banished to Alaska after another disappointing season, are trying to play hockey on a huge plane of glacial ice, tilted at an angle θ to the horizontal as shown. Due to static friction, their puck is initially hanging at rest on the tilted glacier, when a sudden blow from a player's stick projects it horizontally directly across the slope with speed v=8 m/s. The coefficient of kinetic friction between the puck and the surface of the ice is $\mu_k = \tan\theta$. Assuming the glacier is sufficiently long and wide, what is the final steady state speed *w* of the puck? Answer in m/s?

(a) 0
(b) 2
(c) 4
(d) 6
(e) 8



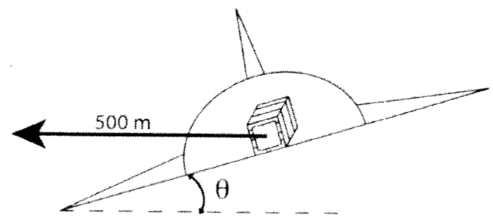
12) BP executives are testing out a new mini robotic submarine in anticipation of their next off-shore oil platform blowout. They take a massless tank of total volume V=0.01 m^3, place the submarine in it, and fill it with water. The flat-bottomed tank sits on a frictionless table. Initially hovering at rest at the center of the tank, the submarine (which is programmed to be wary of the EPA) is spooked by a passerby. It moves to the left and comes to rest again exactly 1 m closer to the left end of the tank. The volume of the submarine is 0.01V, the density of water 1000 kg/m^3, and the density of the submarine is 2000 kg/m^3. What is the distance (in cm) that the tank has moved along the table to the right?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4



1)A large refrigerated cargo plane is delivering crates of ice cubes to the arctic as part of the conservative government's Economic Action Plan. The crates sit unsecured in the middle of the flat floor. Near the drop zone, the plane begins flying in a uniform horizontal circle at 150 m/s. If the radius of the circle is 500 m, and the plane's wings are tilted at an angle θ from the horizontal so that a force on the plane is provided by the lift perpendicular to the wing surface, what is the minimum coefficient of static friction required to prevent the crates from sliding?

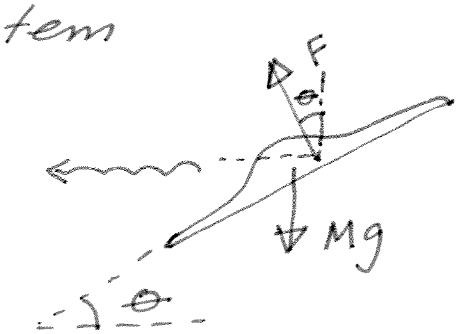
(a) 0.000
(b) 0.110
(c) 0.217
(d) 0.323
(e) 0.434



Let "M" be the total mass

Let "F" be the lift Force.

∴ f.b.d of the system



$\Sigma \vec{F} = m\vec{a} \uparrow$

$F\cos\theta - Mg = 0$

$F\cos\theta = Mg \quad —①$

now $\Sigma \vec{F} = m\vec{a} \leftarrow$

$F\sin\theta = Ma_r = \frac{Mv^2}{R} \quad —②$

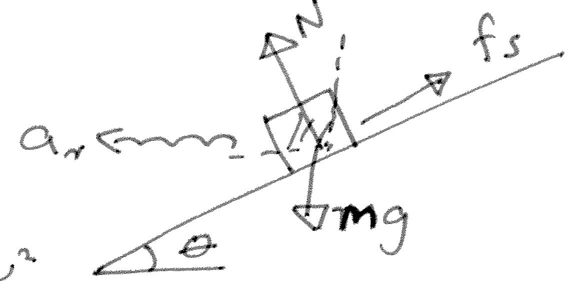
$\frac{②}{①} \quad \tan\theta = \frac{v^2}{Rg} \qquad v = 150\ m/s$ and $R = 500\,m$

$\therefore \theta = 77.7°$ which we can use for the rest of the question

- Continued -

now for the Crate

f.b.d for Crate



$\Sigma \vec{F} = m\bar{a}$ ←

$N \sin\theta - f_s \cos\theta = \frac{mv^2}{R}$

Since the crate is on the verge of sliding

$f_s = \mu_s N$

$\therefore N \sin\theta - \mu_s N \cos\theta = \frac{mv^2}{R}$ — ③

$\Sigma \vec{F} = m\bar{a}$ ↑

$N \cos\theta + f_s \sin\theta - mg = 0$

$N \cos\theta + \mu_s N \sin\theta = mg$ — ④

$\therefore \frac{③}{④} \Rightarrow \frac{\cancel{N} \sin\theta - \mu_s \cancel{N} \cos\theta}{\cancel{N} \cos\theta + \mu_s \cancel{N} \sin\theta} = \frac{v^2}{Rg}$

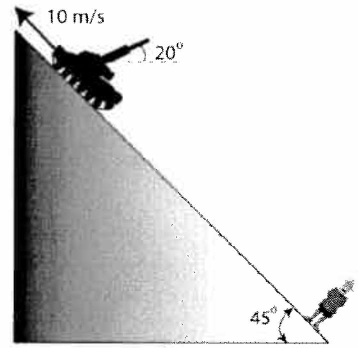
$\frac{\tan\theta - \mu_s}{1 + \mu_s \tan\theta} = \frac{v^2}{Rg} = 4.6$

$\therefore \mu_s = \frac{\tan 77.7 - 4.6}{4.6 \tan 77.7 + 1} = \underline{\underline{0}}$

Hence answer should be (a)

- #1 continued. -

2)Radical Albertan separatists are driving a tank at a constant speed of 10 m/s up a 45 degree mountain slope, in a vain attempt to invade British Columbia. Pursued by the Mounties, the tank fires a 10 kg watermelon, with muzzle velocity 25 m/s, back along its path. Ignore the distance from the muzzle to the slope. If the gun barrel is pointed 20 degrees above the horizontal as shown, how far is the melon from the tank when it finally strikes the slope below? Answer in m.



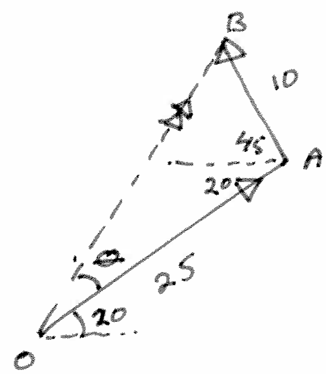
(a) 84.6
(b) 101
(c) 152
(d) 217
(e) 269

Let $\vec{v}_{m,T}$ = velocity of melon relative to the Tank

Let $\vec{v}_{T,E}$ = velocity of the Tank relative to Earth.

$\vec{v}_{m,E}$ = velocity of melon relative to Earth.

now $\vec{v}_{m,E} = \vec{v}_{m,T} + \vec{v}_{T,E} \Rightarrow$



$\vec{OB} \Rightarrow \vec{v}_{m,E}$

using the cosine Law

$$v_{m,E} = \left[25^2 + 10^2 - 2(25)(10)\cos(65)\right]^{1/2}$$

$$= \underline{22.66}\ m/s$$

using the Sine Law on OAB

$$\frac{OB}{\sin 65} = \frac{AB}{\sin\theta} \qquad \therefore \sin\theta = \frac{10\sin 65}{22.66}$$

$$\therefore \theta = \underline{23.57^\circ}$$

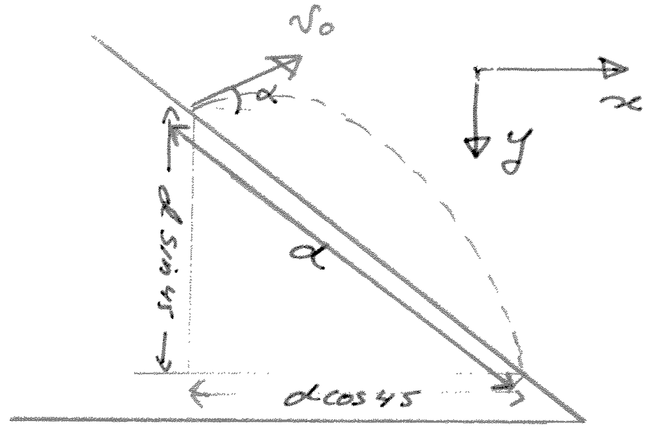
Lets indicate $v_{m,E}$ to be v_0

now we have a projectile motion problem on an incline plane.

- Continued -

now $\alpha = \cancel{\theta} + 20 = 43.57^\circ$

Let x be the horizontal axis



$V_{ox} = V_o \cos\alpha = 22.66 \cos 43.57$
$= 16.41 \text{ m/s}$

and $V_{oy} = V_o \sin\alpha$
$= 15.61 \text{ m/s}$

using $(x - x_0) = V_{ox} t + \frac{1}{2} a_x t^2$
in the horizontal direction $a_x = 0$

$\therefore d \cos 45 = 16.41\, t$

$\therefore t = \dfrac{d \cos 45}{16.41}$

using the same equation in the "y" direction ($\downarrow$ is positive)

$(y - y_0) = V_{oy} t + \frac{1}{2} a_y t^2 \quad \downarrow$

$$d \sin 45 = -15.62 \, \frac{d \cos 45}{16.41} + 4.9 \, \frac{d^2 \cos^2 45}{(16.41)^2}$$

$d = 151.35 \text{ m}$

$\therefore t = 6.52 \text{ sec.}$

now during this time the Tank travels a distance "d'" up the slope

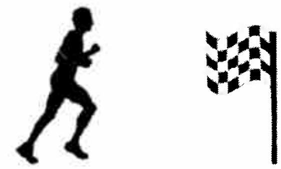
$d' = vt = (10)(6.52) = 65.2 \text{ m}$

$\therefore$ The total separation is $D = d' + d$
$= 65.2 + 151.35$
$= 216.55 \approx \underline{217 \text{m}}$

answer is d .

#2 continued.

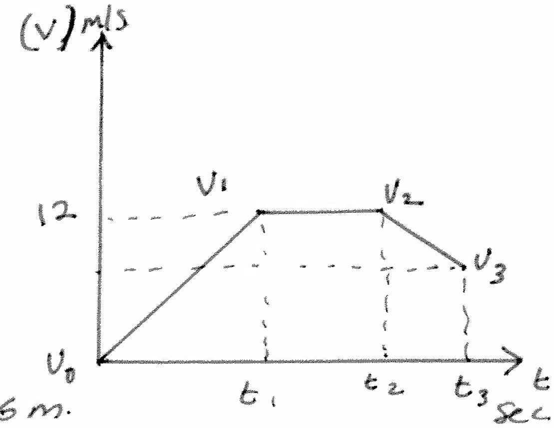
3) Willie in a sprint race,
Started off at a super pace;
Ran out of steam, as others passed.
Yes you've guessed it - he finished last!



A.A.

Willie, in a 100 m race, initially accelerates uniformly from rest at 2.00 m/s^2 until reaching his top speed of 12.0 m/s. He maintains this speed, until he is 16.0 m from the finish line, but then fades and decelerates uniformly, crossing the line with a speed of only 8.00 m/s. What was Willie's total time for the race? Answer in seconds.

(a) 10.8
(b) 11.2
(c) 11.6
(d) 12.0
(e) 12.5



acceleration stage

using $v^2 - v_0^2 = 2a(x - x_0)$

$v = 12 \, m/s$ $\quad v_0 = 0 \, m/s$ and $a = 2 \, m/s^2$

$\therefore 12^2 = 2(2)(x - x_0) \Rightarrow x - x_0 = \underline{x_1 = 36 \, m.}$

using $\bar{v} = \bar{v}_0 + \bar{a}t \Rightarrow 12 = 0 + 2t_1 \quad \therefore \underline{t_1 = 6 \, sec.}$

Constant speed stage

$x_2 = 100 - 36 - 16 = 48 \, m$

$\therefore t_2 = 48/12 = \underline{4 \, sec.}$

Deceleration stage

using $v^2 - v_0^2 = 2a(x - x_0)$

$v^2 = 8^2, \; v_0^2 = 12^2$ and $x - x_0 = 16$

$\therefore a = -2.5 \, m/s^2$

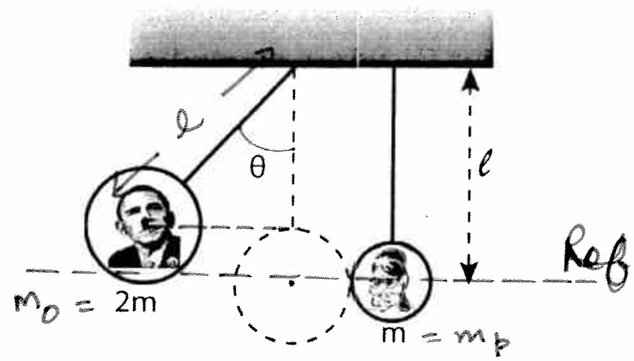
now using $\bar{v} = \bar{v}_0 + \bar{a}t \quad v = 8, \; v_0 = 12$

$8 = 12 - 2.5 t_3 \quad \therefore t_3 = \frac{4}{2.5} = 1.6 \, sec$

$\therefore t_1 + t_2 + t_3 = 6 + 4 + 1.6 = \underline{11.6 \, sec.}$

answer should be (c).

4)Barak Obama proposes a TV discussion with Sarah Palin. They design a special set, where each sits in a plastic sphere (so that their center of mass is at the center of the sphere), suspended at a distance ℓ from the ceiling. The separation at the ceiling keeps both ropes vertical when the spheres contact. Obama's sphere (twice the mass of Palin's, who is a known lightweight) is pulled to the left, and released from θ = 60 deg to the vertical as shown. What maximum angle from the vertical does Palin's sphere reach on her first swing, if the collision is completely elastic? Answer in degrees.



(a) 20.2
(b) 41.8
(c) 62.1
(d) 83.6
(e) Greater than 90 (she hits the ceiling)

let v_o be the speed of obama at Ref

$\therefore$ using conservation of energy.

$$m_o g \, \ell (1-\cos\theta) = \tfrac{1}{2} m_o v^2$$

$$2m \, g \, \ell (1-\cos 60) = \tfrac{1}{2}(2m) v_o^2$$

$$\therefore v_o^2 = 9.8\,\ell \quad \text{— ③}$$

using conservation of momentum $\sum \vec{p_i} = \sum \vec{p_f} \rightarrow$

$$2m v_o = 2m v_o' + m v_p$$

$$2v_o = 2v_o' + v_p \quad \text{——— ①}$$

Since it is an elastic collision, the energy is conserved (kinetic energy).

$$\therefore \tfrac{1}{2}(2m)v_o^2 = \tfrac{1}{2}(2m)v_o'^2 + \tfrac{1}{2} m v_p^2$$

$$\therefore 2v_o^2 = 2v_o'^2 + v_p^2$$

$$2(v_o^2 - v_o'^2) = v_p^2$$

$$2(v_o + v_o')(v_o - v_o') = v_p^2$$

from ① $\quad 2(v_o - v_o') = v_p$

$$\therefore v_p(v_o + v_o') = v_p^2 \quad \text{—— ②}$$

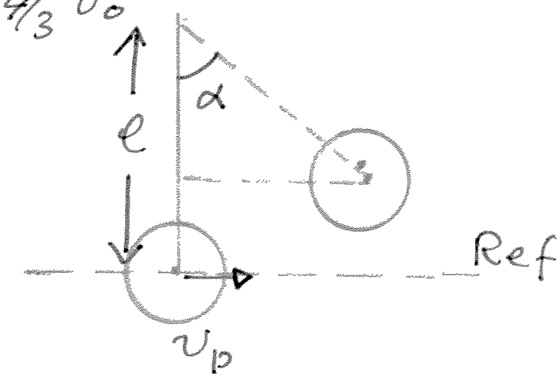
$\therefore$ sub into ① $\Rightarrow$ $2v_o = 2v_o' + v_o + v_o'$

$$\therefore v_o' = v_o/3$$

- Continued -

now using (2)

$$v_p = v_0 + v_0/3 = 4/3\, v_0$$



using conservation of energy for Palin

$$\frac{1}{2} m_p v_p^2 = m_p g l (1-\cos\alpha)$$

$$\frac{1}{2} \cancel{m_p} \frac{16}{9} v_0^2 = \cancel{m_p} g l (1-\cos\alpha)$$

but from (3) $v_0^2 = g l$

$$\therefore \frac{1}{2} \frac{16}{9} \cancel{gl} = \cancel{gl} (1-\cos\alpha)$$

$$\therefore \frac{16}{18} = 1-\cos\alpha$$

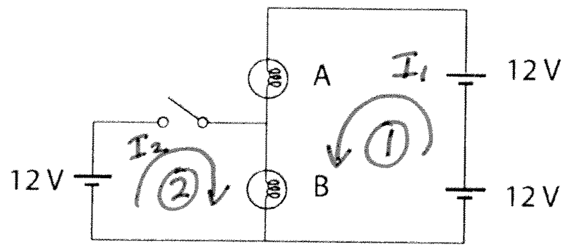
$$\therefore \cos\alpha = \frac{2}{18}$$

$$\therefore \alpha = 83.6^\circ$$

answer (d)

#4 Continued.

5)The light bulbs in the circuit pictured below are identical. When the switch is closed:



(a) Both lights go out.
(b) One of the lights goes out.
(c) The intensity of light bulb A increases.
(d) The intensity of light bulb B increases.
(e) Nothing changes.

The simplest way to look at this is to realize that the potential difference across each bulb is 12V, when the switch is open. Due to the polarity of the cells, closing the switch will not change potential difference. Hence there will be NO change in current. ∴ the intensity will remain the same.

OR. using Kirchhoff's Law

for loop ① ⇒ $24 - I_1 R - (I_1 + I_2) R = 0$ — ①

for loop ② $-(I_2 + I_1) R + 12 = 0$ — ②

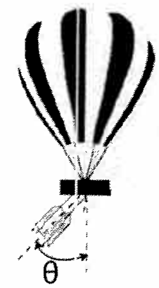
∴ from ① ⇒ $I_1 R = 12$

② ⇒ $I_1 R + I_2 R = 12$

$\therefore I_2 = 0$

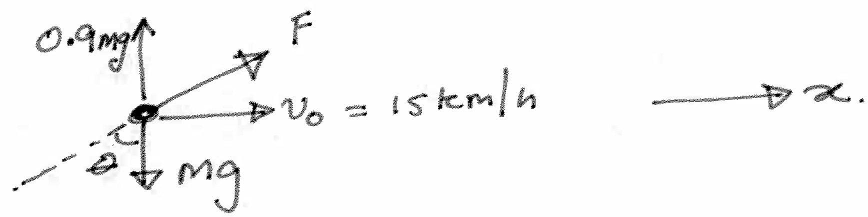
6)Peter Mackay, learning from Fox News that the Russians are en-route to claim the North Pole, commandeers a hot air balloon to confront them. While cruising with the polar airflow at an altitude of 1 km with a constant velocity of 15 km/h, the temperature in the balloon suddenly drops, reducing its buoyant force by 10%. Fortunately, he has a jet system that can create a thrust, so that, if pointed directly down, it would exactly compensate for the lost buoyancy. In order to claim the Pole, Mackay needs to travel an additional horizontal distance of 2 km within 2 minutes, and a quick calculation shows that he won't make it without assistance. He sets the angle θ equal to 30 degrees and turns on the jet. What will his altitude be as he passes over the Pole? Neglect any air resistance.

(a) He would pass over the Pole too late.
(b) He will hit the ground too soon and be eaten by a polar bear.
(c) 556 m
(d) 471 m
(e) 293 m



Let "F" be the force

F.b.d.



$\Sigma \vec{F} = ma_x \rightarrow$

$F \sin\theta = ma_x$

$F = 0.1\,mg$ $\qquad \theta = 30^\circ$

$\therefore (0.1)\,mg \sin\theta = ma_x.$

$\therefore \quad a_x = 0.49\ m/s^2.$

$\Sigma F = ma_y \downarrow$

$mg - 0.9mg - 0.1\,mg \cos 30 = ma_y$

$\therefore a_y = 0.13\ m/s^2 \downarrow$

now for horizontal motion

$15\ km/hr = 4.16\ m/s$ ~

$\therefore (x - x_0) = V_{0x} t + \frac{1}{2} a_x t^2 \rightarrow$

$2000 = 4.16t + \frac{1}{2}(0.49)\,t^2.$

$\therefore \quad 0.245t^2 + 4.16t - 2000 = 0$

we have a quadratic. -Continued-

$$\therefore\ t = \frac{-4.16 \pm \sqrt{(4.16)^2 - 4(0.245)(-2000)}}{0.49}$$

from which we take the positive value of "t"

$t = 82.25$ sec.

$\therefore$ the vertical drop during this time

$$(y - y_0) = V_{oy}t + \tfrac{1}{2}a_y t^2 \downarrow$$

$$= 0 + \tfrac{1}{2}(.13)(82.25)^2$$

$$= 439.8 \text{ m.}$$

$\therefore$ the altitude $1000 - 439.8$

$\simeq 560$ m

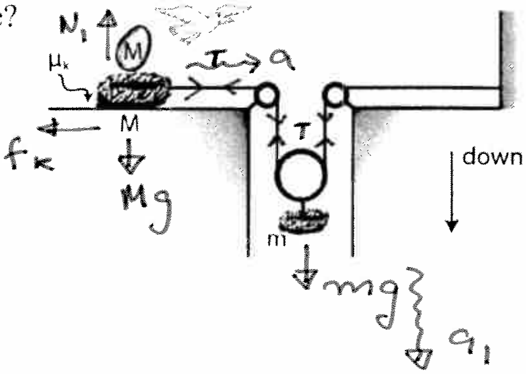
$\simeq 556$ m.

$\therefore$ answer C

#6 continued.

7)A bird watcher sets up an experiment with two bird nests, mass M and m, which are attached to a system of frictionless massless pulleys via a single rope, and released as shown. Before her second trial, a bird lays an egg into one nest, doubling its mass from M to 2M. If the sliding nest's acceleration decreases from g/2 to g/4 between the two trials, what is the coefficient of kinetic friction μ_k between the nest and the horizontal plane?

(a) 0.10
(b) 0.26
(c) 0.35
(d) 0.60
(e) 0.72



for M, $\Sigma \vec{F} = ma \rightarrow$

$T - f_k = Ma$ — ①

$\Sigma \vec{F} = m\bar{a}$ for m $\downarrow$

$mg - 2T = ma_1$

Conservation of string $\Rightarrow a = 2a_1$

$\therefore a_1 = a/2.$

$\therefore 2mg - 4T = ma$ — ②

now ② + 4 × ① $\Rightarrow 2mg - 4f_k = (4M + m)a$

$f_k = \mu_k N_1 = \mu_k Mg$

$\therefore 2mg - 4\mu_k Mg = (4M + m)a.$

it is said that $a = g/2.$

$\therefore 2mg - 4\mu_k Mg = 4M\,g/2 + mg/2.$

$3/2\, m = M(2 + 4\mu_k)$ — ③

Doubling the mass decreases the acceleration to g/4.

$\therefore 2mg - 8\mu_k Mg = (8M + m)\,g/4$

$\frac{7}{4} m = M(2 + 8\mu_k)$ — ④

$\therefore \frac{③}{④} \Rightarrow 0.86 = \frac{2 + 4\mu_k}{2 + 8\mu_k}.$

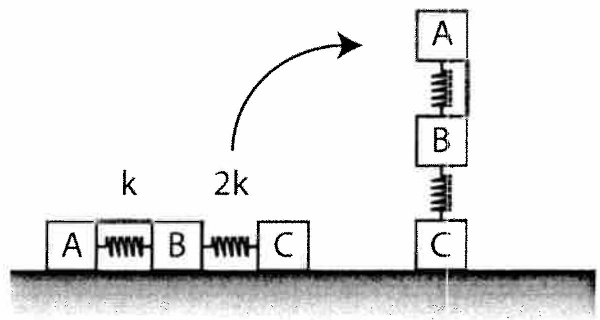
$1.71 + 6.88\mu_k = 2 + 4\mu_k.$

$\therefore 2.88\mu_k = 0.29$

$\therefore \mu_k = 0.1$ answer: a

8)Michael Ignatieff and Jack Layton decide to see if Jim Flaherty can calculate a decent budget. First, they bring three blocks each of mass 0.5 kg and connect them in line to one another on a horizontal table using stiff springs. The springs, with constants k=1000 N/m and 2k, are at their equilibrium lengths. Next, blocks A and B are connected via a massless string which is tightened to a force of F=20 N. Finally the whole system is tipped upright onto block C as shown in the figure. They ask Flaherty what the total change in the distance between blocks A and C is in going from the original horizontal to the final vertical configuration. What should his answer be in cm?

(a) 0.50
(b) 1.50
(c) 2.00
(d) 2.25
(e) 2.50



When a force of 20N is applied between A & B

$\Sigma \vec{F} = k\bar{x}$

$20 = 1000\,x \quad \therefore x = 0.02\text{ m} = 2\text{ c.m.}$

when in vertical position $M_A g = (0.5)(9.8) = 4.9\text{N}$

$\therefore$ The spring force is greater than the gravitation force of A. $\therefore$ This spring will Not compress more!

now A and B will compress the spring of

$k = 2000$ N/M.

$\Sigma \vec{F} = ky \Rightarrow (0.5 + 0.5)9.8 = ky = 2000\,y$

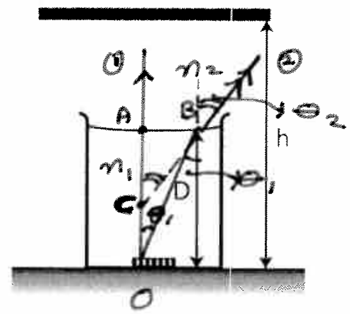
$\therefore y = 0.0049$ m or 0.5 CM.

$\therefore$ The total deviation is $2 + 0.5 = 2.5$ CM.

answer is (e)

9)Cheech finds his favourite drinking glass and decides to place a coin at the bottom of it. He then places a flat mirror a distance h above the base and asks Chong to predict the position of the image of the coin. Chong accepts the challenge but now fills the glass to a height D with his favourite drink (index of refraction 1.33) and challenges Cheech to predict the distance from the base of the glass to the image of the coin in the mirror. Assuming the small-angle approximation, what would Cheech's correct answer be?

(a) h-D/2
(b) 2h-D/4
(c) h-D/4
(d) h-D
(e) 2h-D



Ray ① goes undeviated.

Ray ② will bend at B

The virtual image will be formed at "C", the point where the 2 refracted rays appear to meet.

applying Snell's Law at B

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $\qquad n_1 = 1.33 = 4/3$ and $n_2 = 1$

for small angles $\sin\theta = \tan\theta$

$\therefore n_1 \tan\theta_1 = n_2 \tan\theta_2$

$n_1 \frac{\cancel{AB}}{OA} = n_2 \frac{\cancel{AB}}{AC}$

$\therefore AC = n_2 \frac{OA}{n_1} = \frac{OA}{n_1} = \frac{3}{4} D$

$\therefore OC = \frac{1}{4} D$ $\qquad \therefore$ Image of "C" on the Mirror will be $h - \frac{1}{4}D$ behind the flat mirror.

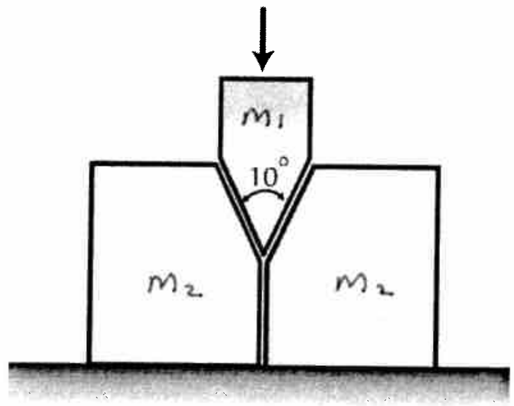
$\therefore$ from the base of the glass it would be

$h + h - \frac{1}{4}D = 2h - D/4$

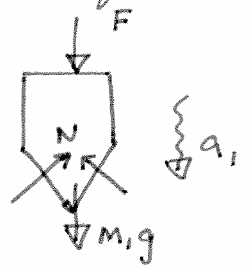
$\therefore$ answer is (b)

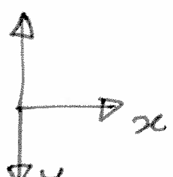
10)Romeo tries to impress Juliet by showing off what he created in shop class. He has machined two large blocks, each of mass 1500 kg, with a precise angle as shown in the diagram. He puts them on a very smooth surface, and on top places a vertical 10 degree symmetric wedge of mass 5 kg. He then asks Juliet to apply a gentle force on the wedge, so as to accelerate the two blocks sideways at 0.5 m/s^2. How much force (in Newton's) does Juliet apply? Assume that there is no friction anywhere.

(a) 45.0
(b) 62.0
(c) 75.2
(d) 82.5
(e) 111

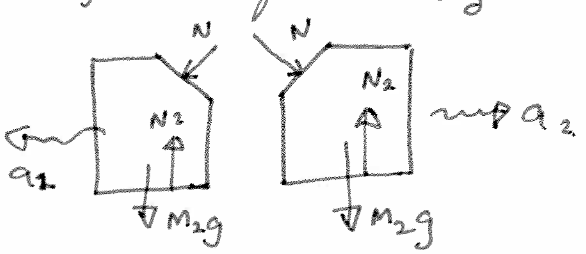


f.b.d for m_1





f.b.d for m_2



$\Sigma F_y = m a_y \downarrow$ for m_1

$m_1 g + F - 2N \sin 5^\circ = m_1 a_1$ — (1)

$\Sigma F_x = m a_x$ for $m_2 \rightarrow$

$N \cos 5^\circ = m_2 a_2 \Rightarrow N = \dfrac{m_2 a_2}{\cos 5^\circ}$ — (2)

using the shape of the objects a_1 and a_2 are related

$\tan 5^\circ = a_2/a_1$ — (3)

now substitute (2) and (3) into (1)

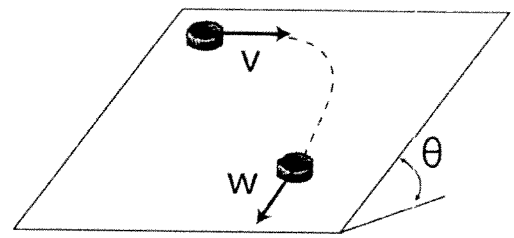
$F = \dfrac{m_1 a_2}{\tan 5} + 2\dfrac{m_2 a_2}{\cos 5^\circ}\sin 5 - m_1 g \Rightarrow 28.5 + 131.23 - 49 = 110.8$

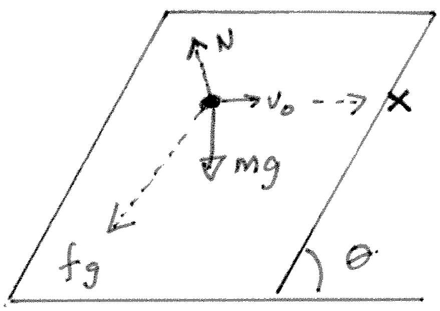
$\therefore F \approx 111$ N

answer (e)

11)The Toronto Maple Laughs, banished to Alaska after another disappointing season, are trying to play hockey on a huge plane of glacial ice, tilted at an angle to the horizontal as shown. Due to static friction, their puck is initially hanging at rest on the tilted glacier, when a sudden blow from a player's stick projects it horizontally directly across the slope with speed v=8 m/s. The coefficient of kinetic friction between the puck and the surface of the ice is $\mu_k = \tan\theta$. Assuming the glacier is sufficiently long and wide, what is the final steady state speed w of the puck? Answer in m/s?

(a) 0
(b) 2
(c) 4
(d) 6
(e) 8





$\sum \vec{F} = m\vec{a}$ perpendicular to the incline

$N - mg\cos\theta = 0$

$\therefore N = mg\cos\theta$

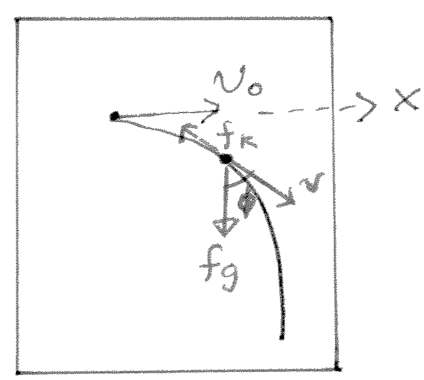
$\therefore f_k$, kinetic friction $= \mu_k N = \mu_k mg\cos\theta = \tan\theta\, mg\cos\theta$

$\therefore f_k = mg\sin\theta$

The component of the gravitational force along the incline $f_g = mg\sin\theta$.

now consider The The Motion along the incline plane

"ϕ" changes with time



Note:- Even though the initial velocity is in the horizontal ("X") direction, due to friction with time the speed will reduce to zero. $\therefore$ The final velocity will be in the "$\vec{f_g}$" direction.

— continued —

at a certain instant if we consider the direction of v to be our positive "x" direction (Not "X").

$$\Sigma F_x = m a_x \Rightarrow f_g \cos\phi - f_k = m a_x \quad \text{— ①}$$

$$\text{and} \quad \Sigma F_y = m a_y \Rightarrow f_g - f_k \cos\phi = m a_y \quad \text{— ②}$$

now a_x and a_y are instantaneous accelerations.

$$f_g = f_k = mg \sin\theta$$

$$\therefore \quad ① + ② \Rightarrow \quad m(a_x + a_y) = 0$$

$$\Rightarrow m\left(\frac{dv_x}{dt} + \frac{dv_y}{dt}\right) = 0$$

which can be written as $m \dfrac{d}{dt}\left[v_x + v_y\right] = 0$

now "m" is not zero $\therefore \dfrac{d}{dt}\left[v_x + v_y\right] = 0$

$\therefore v_x + v_y = \text{constant}$, for any instant

at $t = 0$ $\quad v_y = 0$ and $v_x = v_0 = 8$ m/s

$$\therefore v_x + v_y = 8 \quad \text{— ③}$$

at steady state at some time T

v_x and v_y will be in the direction of "f_g".

$\therefore$ at $t = T$ $\quad v_x = w$ and $v_y = w$

$$\therefore 2w = \text{const} = 8$$

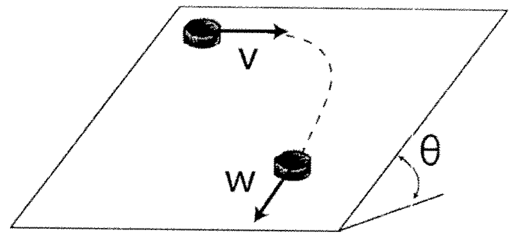
$$\therefore w = 4 \text{ m/s.}$$

answer ©

#11 continued

11)The Toronto Maple Laughs, banished to Alaska after another disappointing season, are trying to play hockey on a huge plane of glacial ice, tilted at an angle to the horizontal as shown. Due to static friction, their puck is initially hanging at rest on the tilted glacier, when a sudden blow from a player's stick projects it horizontally directly across the slope with speed v=8 m/s. The coefficient of kinetic friction between the puck and the surface of the ice is $\mu_k = \tan\theta$. Assuming the glacier is sufficiently long and wide, what is the final steady state speed w of the puck? Answer in m/s?

(a) 0
(b) 2
(c) 4
(d) 6
(e) 8



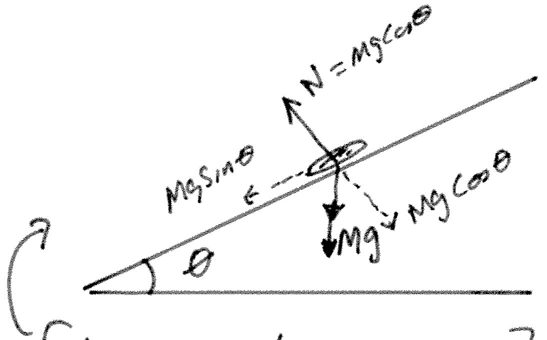
This Solution keeps away from the "language" of Calculus and uses only $\Sigma \vec{F} = m\bar{A}$, which says that an object of mass M subject to a total force $\vec{F}$ will have an acceleration $\bar{A}$. We agree that by definition, $\bar{A}$ is the rate of change of $\vec{v}$ with respect to time, written

$\bar{A} = \frac{\Delta \bar{V}}{\Delta t}$, or "$\frac{\text{change in } V}{\text{change in } t}$".

This is a vector equation which can be applied in <u>any direction</u> and Δt is as small as we wish.

After the blow from the stick, the forces acting on the puck are, Gravitational weight and both Normal and Frictional forces by ice

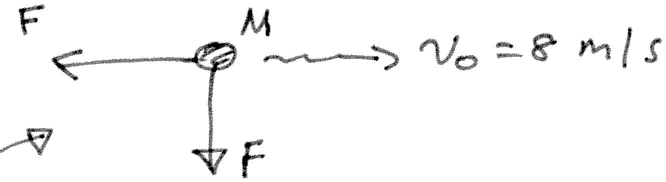
— Continued —



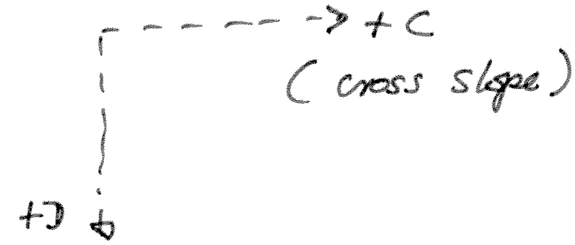
[looking horizontal across the slope]

[The friction force
$R = \mu_k N = \frac{\sin\theta}{\cos\theta} Mg\cos\theta$
$= mg\sin\theta.$
This acts into the page since the puck moves out]

((Let "$mg\sin\theta$" = "F" for Simplicity))



[looking down at the ice surface just after the slap]



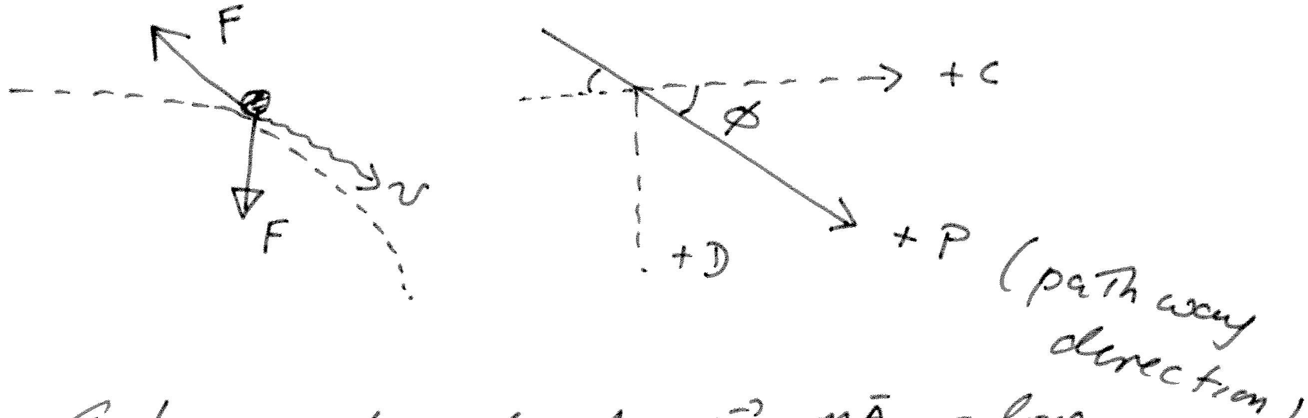
use these "direction axis" initially

So after the first short "Δt" interval the "C" component will be smaller than 8 by $F\Delta t$ and the "D" component will be larger than 0, by the same amount.

So the puck will veer a bit to its right and trace some sort of curved pathway.

— #11 continued —

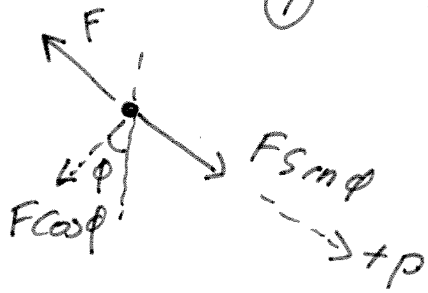
Let us draw the puck at some later time, where the path has swung down through an angle ϕ



Take components of $\Sigma \vec{F} = M\vec{A}$ along the +P axis direction.

$$F \sin\theta - F = M A_P$$

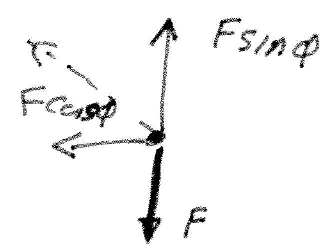
(1) $$M \Delta V_P = F[\sin\phi - 1]$$



now take components of $\Sigma \vec{F} = M\vec{A}$ along the +D axis direction

$$F - F\sin\phi = M A_D$$

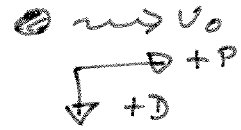
(2) — $$M \Delta V_D = F(1 - \sin\phi)$$



So we see that for <u>any</u> small Δt the corresponding changes in ΔV_P and ΔV_D are exactly equal and opposite.

so $\Delta [V_P + V_D] = 0$

$\therefore V_P + V_D = \text{Constant}$

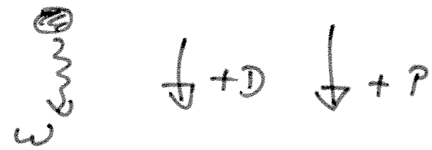


At The start $V_P = 8$ and $V_D = 0$

$\therefore V_P + V_D = 8$

at The end

$V_P = w$ and $V_D = w$



$\therefore V_P + V_D = w + w = 2w$

$\therefore 2w = 8$

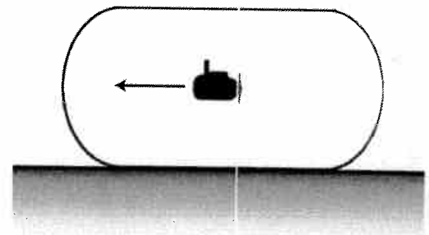
$\therefore w = 4.$

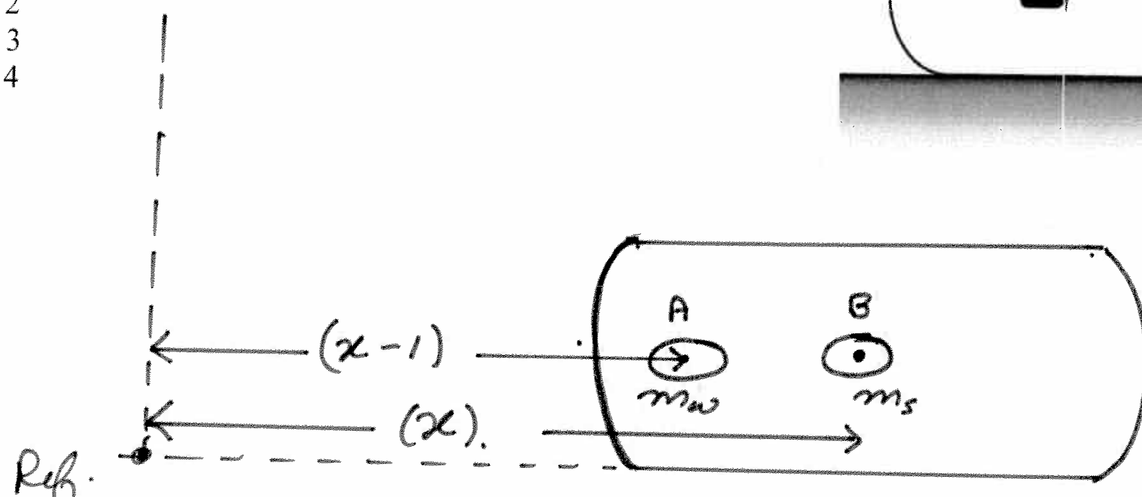
<u>Note</u> at The end $\varphi = 90^\circ$

and at start $\varphi = 0$

12)BP executives are testing out a new mini robotic submarine in anticipation of their next off-shore oil platform blowout. They take a massless tank of total volume V=0.01 m^3, place the submarine in it, and fill it with water. The flat-bottomed tank sits on a frictionless table. Initially hovering at rest at the center of the tank, the submarine (which is programmed to be wary of the EPA) is spooked by a passerby. It moves to the left and comes to rest again exactly 1 m closer to the left end of the tank. The volume of the submarine is 0.01V, the density of water 1000 kg/m^3, and the density of the submarine is 2000 kg/m^3. What is the distance (in cm) that the tank has moved along the table to the right?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4





The massless tank has a volume "V". When filled with water it will have a center of mass at position "B". The submarine too has its center of mass initially at B, and a small volume of .01V.

Position "A" is surrounded by a volume .01V (same as the submarine). This is a problem where the submarine moves to "A" from "B" and the volume 0.01V of water moves to "B" from "A". We will assume the center of mass of the tank filled with water is at "B".

m_w = mass of water volume 0.01V at A

m_s = mass of submarine.

also mass = (volume)(volume density)

m = V ρ

— continued —

In this problem since we do not have any external forces, the linear momentum is conserved.

$$\Sigma \vec{p_i} = \Sigma \vec{p_f}$$

also $M\vec{V}_{CM} = \Sigma \vec{p_i}$ where $\vec{V}_{CM}$ = velocity of the center of mass and M is the total mass.

since $(V_{CM})_i = 0$ $\quad M(V_{CM})_i = \Sigma p_i = 0$

since the motion we are considering is along the x axis $\quad V_{CM} = \dfrac{\Delta X_{CM}}{\Delta t} = 0$

$$\therefore \Delta x_{CM} = 0 \Rightarrow (x_{CM})_i = (x_{CM})_f$$

$$(X_{CM})_i = \frac{\Sigma m_i \bar{x}_i}{\Sigma m_i} = \frac{m_w(x-1) + m_s(x) + Mx}{M + m_s + m_w}$$

$$= \frac{(0.01)V\rho_w(x-1) + (0.01)V\rho_s x + V\rho x}{M + m_s + m_w}$$

$$= \frac{(1000)(0.01)V(x-1) + (2000)(0.01)Vx + (1000)Vx}{M + m_s + m_w}$$

$$= \frac{(1030x - 10)V}{M + m_s + m_w}$$

now when the submarine moves 1 m to the left the tank will move some distance "d" to the right. $\therefore$ the new center of mass from our reference point will be $(X_{CM})_f$.

$$(X_{CM})_f = \frac{\Sigma m_i \bar{x}_i}{\Sigma m_i} = \frac{m_s(x-1+d) + m_w(x+d) + M(x+d)}{M + m_s + m_w}$$

(#12 continued)

(2)

$$(X_{cm})_f = \frac{(2000)(0.01)V(x-1+d) + (1000)(0.01)V(x+d) + 1000V(x+d)}{M + m_s + m_w}$$

$$= \frac{(1030x + 1030d - 20)V}{M + m_s + m_w}$$

Since $(X_{cm})_i = (X_{cm})_f$.

$$\cancel{1030x} - 10 = \cancel{1030x} + 1030d - 20$$

$$\therefore 1030d = 10$$

$$\therefore d = \underline{\underline{0.00971\ m}}.$$

$$\therefore d \approx .971\ cm = \underline{\underline{1\ cm}}.$$

answer (b)