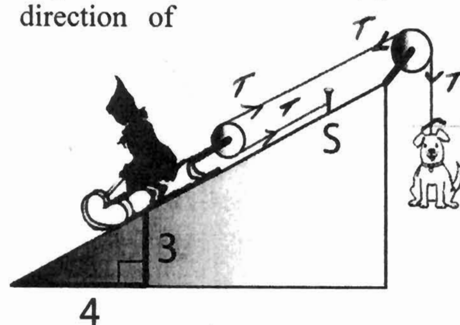
1) Little Orphan Annie and her dog Sandy are in trouble on a frictionless snow covered slope in Northern Canada as shown. Annie and the sled have a mass of 65.0 kg. A strong massless rope is tied to a stake at S, passes down the slope, around a massless pulley whose axle pulls on the sled, back up along the slope to a second massless pulley whose axle is fixed at the edge of a cliff and finally vertically down to Sandy, who has a mass of 45.0 kg and keeps a firm grip on it with his teeth. Calculate the magnitude of the acceleration of the sled along the slope. Answer in terms of gravitational acceleration, g. (For full marks in hand grading indicate also the direction of acceleration.)

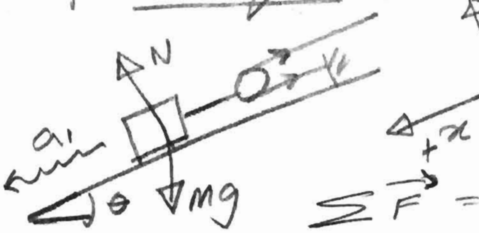


(a) 0.182
(b) 0.208
(c) 0.329
(d) 0.437
(e) 0.545

Let Annie be m_1 and Sandy be m_2.

Since we have only one string the tension every where will be "T".

f.b.d for m_1

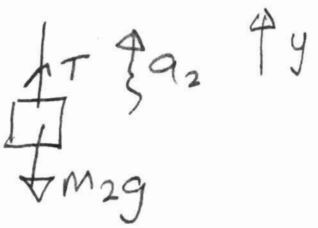


let +x be along the incline pointing down as shown.

$\sum \vec{F} = m\vec{a}$ (along the +ve x axis)

$$mg\sin\theta - 2T = m_1 a_1 \quad \text{—①}$$

fbd for Sandy



$\sum F_y = ma_y$ ↑

$T - m_2 g = m_2 a_2$.

However due to "conservation of string"

$a_2 = 2a_1$

$$\therefore T - m_2 g = 2ma_1 \quad \text{—②}$$

now ① + 2② $\Rightarrow$ $m_1 g\sin\theta - 2m_2 g = (m_1 + 4m_2)a_1$

$$\therefore a_1 = \frac{(m_1\sin\theta - 2m_2)g}{m_1 + 4m_2}$$

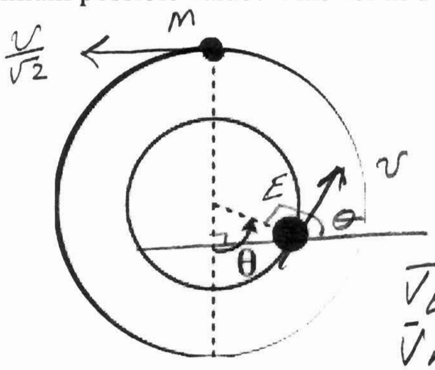
$$= \frac{[65(3/5) - 2(45)]g}{245} = -0.208g$$

Hence our direction is pointing opposite to the side we chose.

answer (b)

2) Suppose Earth and Mars both move on concentric counter clockwise circular orbits, with radii R and 2R and periods T and, respectively. For what angle θ, in the range of 0 to 180 degrees, will the magnitude of the relative velocity of the planets be exactly one half the maximum possible value? Answer in degrees.

(a) 37
(b) 81
(c) 107
(d) 123
(e) 147



Let E ⇒ Earth
M ⇒ Mars
S ⇒ Stationary reference frame

$\vec{V}_{E,S}$ = Velocity of Earth relative to S
$\vec{V}_{M,S}$ " " Mars " " "

$$|\vec{V}_{E,S}| = V_{ES} = \frac{2\pi R}{T} = v$$

$$|\vec{V}_{M,S}| = V_{M,S} = \frac{2\pi(2R)}{2\sqrt{2}\,T} = \frac{v}{\sqrt{2}}.$$

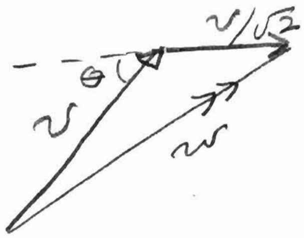
$$\vec{v}_{E,M} = \vec{v}_{E,S} + \vec{v}_{S,M}$$

$$\vec{v}_{S,M} = -\vec{v}_{M,S}$$

∴ $\vec{V}_{ES}$ = v at angle θ and $\vec{v}_{S,M}$ = $\xrightarrow{v/\sqrt{2}}$

using graphical methods

$\vec{v}_{E,M} = \vec{v}_{E,S} + \vec{v}_{S,M}$ ⇒



using the Cosine Rule

$$w^2 = v^2 + \left(\frac{v}{\sqrt{2}}\right)^2 \pm 2\, v \frac{v}{\sqrt{2}} \cos(180-\theta).$$

$$w^2 = \frac{3}{2}v^2 + \sqrt{2}\, v^2 \cos\theta. \quad \text{—— (1)}$$

This becomes a maximum when $\theta = 0$

$$\therefore w_{max}^2 = v^2\left(\frac{3}{2} + \sqrt{2}\right) \quad \text{—— (2)}$$

We need to find θ when $w = \frac{w_{max}}{2}$.

$$\therefore \frac{w^2}{4} = v^2\left(\frac{3}{2} + \sqrt{2}\cos\theta\right)$$

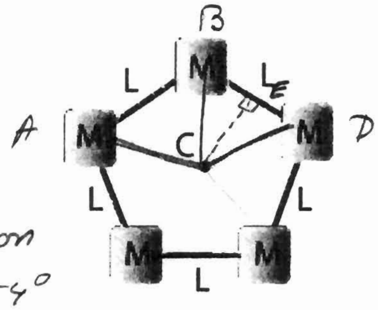
$$\frac{v^2}{4}\left(\frac{3}{2} + \sqrt{2}\right) = v^2\left(\frac{3}{2} + \sqrt{2}\cos\theta\right)$$

$$\frac{\frac{3}{8} + \frac{\sqrt{2}}{4} - \frac{3}{2}}{\sqrt{2}} = \cos\theta \Rightarrow \theta = 123 \text{ or}$$

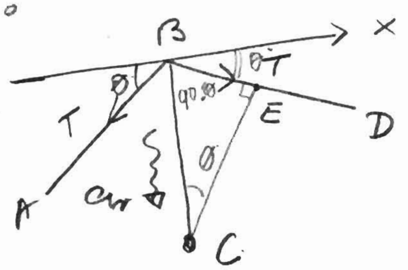
∴ $\theta = 123°$ answer d

3) Your Physics Class built a model of our central government having five political leaders, each represented by a massive block of ice (mass M) and joined at its centre to two identical neighbours by massless ropes (length L) as shown. This forms a closed ring resting on a frictionless frozen horizontal lake. Initial use of blasts from air hoses sets up a uniform circular motion of the entire ring about the central point C, but from then on it coasts around at a constant rate of exactly two revolutions per minute. The only horizontal forces acting are the tensions in the five ropes. Given that L = 123 m and M = 234 kg, calculate the magnitude of the tension force in each rope. Answer in N.

(a) 540
(b) 865
(c) 914
(d) 1080
(e) 1320



Since we have a Pentagon
$A\hat{B}D = 108°$ and $A\hat{B}C = 54°$
$B\hat{C}D = 72°$
The perpendicular drawn from C to BD is CE
it can be shown that $B\hat{C}E = \frac{72}{2} = 36°$ and $BE = ED = L/2$
Hence if we take $B\hat{C}E = \theta = 36°$
now using the triangle BCE



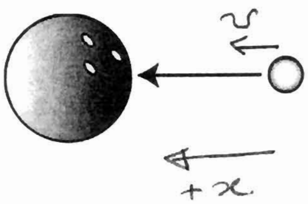
$\sin\theta = \frac{L/2}{R}$ ∴ $R = \frac{L}{2\sin\theta}$
$(BC = R)$

now if the system is rotating about C at 2 revolutions/m
each mass will have a speed of $\frac{2\pi R}{30}$ m/s
$\sum \vec{F} = m a_r$ in the radial direction

$$2T\sin\theta = m a_r = \frac{mv^2}{R} \Rightarrow T = \frac{mv^2}{2R\sin\theta}$$
$$= \frac{m}{R}\left(\frac{2\pi R}{30}\right)^2 \frac{1}{2\sin\theta}$$
$$= \left(\frac{\pi^2}{900}\right)\left(\frac{mL}{\sin^2\theta}\right)$$
$$= 914 \text{ N}$$

answer (c)

4) A ping pong ball is fired at a bowling ball initially at rest and bounces back elastically. Compared to the bowling ball, the ping pong ball after the collision has



(a) greater magnitude momentum but less kinetic energy
(b) greater magnitude momentum and more kinetic energy
(c) lower magnitude momentum and less kinetic energy
(d) lower magnitude momentum but more kinetic energy
(e) none of the above

Let the mass of ping pong ball be m and bowling ball be M. If the initial velocity of the ping pong ball v

$\Sigma \vec{p}_i \leftarrow = mv$

after collision let the ping pong ball reverse its direction v' and bowling ball move in the forward direction V'

$\therefore \Sigma p_i = \Sigma p_f \leftarrow \Rightarrow mv = -mv' + MV'$

$\therefore m(v+v') = MV'$ — ①

Conservation of energy $\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}MV'^2$

$m(v^2 - v'^2) = MV'^2$

$\therefore m(v+v')(v-v') = MV'^2$ — ②

from ① & ② $MV'(v-v') = MV'^2$

$\therefore v - v' = V'$ — ③ sub this back into ①

$mv + mv' = Mv - Mv' \Rightarrow v' = \left(\frac{M-m}{M+m}\right)v$ & $V' = \frac{2mv}{M+m}$

$M \gg m \quad \therefore M+m \approx M$

$\therefore p_{bowling} = MV' \approx \frac{M\,2mv}{M} \approx 2mv$

$p_{ping\,pong} = mv' \approx m\left(\frac{M-m}{M}\right)v \approx m\left(\frac{M}{M} - \frac{m}{M}\right)v$

since $M \gg m \quad \frac{m}{M} \to 0 \quad \therefore p_{ping\,pong} \approx m(1-0)v \approx mv$

$\therefore p_{bowling} > p_{ping\,pong}$

now lets check the kinetic energy

$(KE)_{ping\,pong} = \frac{1}{2}mv'^2$

$= \frac{1}{2}m\left[\frac{M}{M} - \frac{m}{M}\right]^2 v^2 = \frac{1}{2}m[1-0]v \approx \frac{1}{2}mv^2$

$(KE)_{bowling} = \frac{1}{2}MV'^2 = \frac{1}{2}M\frac{2\cdot 4m^2v^2}{M^2} = \frac{2m^2v^2}{M} = 2\left(\frac{m}{M}\right)mv^2$

now $\frac{m}{M} < 0.1 \quad \therefore (KE)_{bowling} < 0.2mv^2 \quad \therefore (KE)_{ping\,pong} > (KE)_{bowling}$

$(KE)_{ping\,pong} \approx 0.5mv^2$

$\therefore$ answer (d)

5) Little Willie jumped on his sled,
Across the ice with glee he sped;
Hit Millie's sled, which was at rest;
Caused her to yell "Get lost, you pest!"

The collision was linear, with Willie's sled's front hitting the other sled's rear, and since it was completely inelastic, the sleds remain together and move off with a common velocity. If the combined mass of Willie and his sled is one half that of Millie and her sled, find the fraction of the original kinetic energy which was dissipated in the collision.

(a) 1/3 (b) 2/5 (c) 1/2 (d) 2/3 (e) 3/4

m_ω = mass of Willie, v_ω speed of Willie
m_M = mass of Millie v_m speed of Millie
Conservation of momentum ⟶

$$m_\omega v_\omega = (m_\omega + m_m) V \Rightarrow V = \frac{m_\omega v_\omega}{m_\omega + m_m}$$

$$\text{Initial } (KE) = \frac{1}{2} m_\omega v_\omega^2$$

$$\text{final } (K.E) = \frac{1}{2}(M_m + M_\omega) V^2 = \frac{1}{2}\left(\frac{M_\omega^2}{M_m + M_\omega}\right) v_\omega^2$$

$$\therefore \text{ dissipated energy} = (KE)_{final} - (KE)_{initial}$$
$$= \frac{1}{2} M_\omega v_\omega^2 \left[1 - \frac{M_\omega}{M_\omega + M_M}\right]$$

If mass of Willie = $M = M_\omega$
mass of Millie = $2M = M_m$

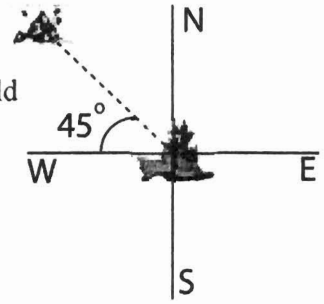
$$\therefore \text{ fraction of dissipated energy} = \frac{\frac{1}{2} M_\omega v_\omega^2 \left[1 - \frac{M_\omega}{M_\omega + M_M}\right]}{\frac{1}{2} M_\omega v_\omega^2}$$

$$= 1 - \frac{M}{3M}$$

$$= \frac{3M - M}{3M} = \frac{2}{3}$$

answer (d)

6) A Canadian Navy icebreaker is headed for Hans island, 32.0 km northwest (NW) of its current position, for a confrontation with its Danish occupiers, when it is suddenly engulfed in heavy fog. The captain maintains a compass bearing of northwest and a speed of 10.0 km/h relative to the water. The fog lifts 3.0 hours later, and the captain notes that he is now exactly 4.0 km south of Hans island (due to constant water current). What compass bearing should the captain have maintained, to reach his destination along a straight course?



(a) 28.6° west of north
(b) 35.2° west of north
(c) 39.6° west of north
(d) 43.1° west of north
(e) 50.0° west of north

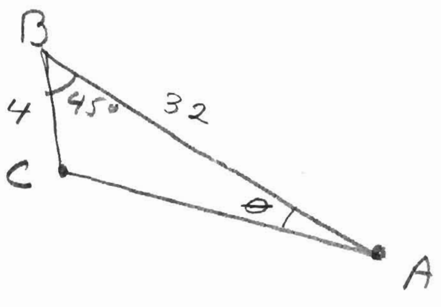
There are many ways of solving this. I will use the more traditional method.

I will call the Icebreaker the "boat".

$\vec{V}_{B,E}$ = Velocity of boat relative to Earth or stationary reference frame.

$V_{R,E}$ = Velocity of River or water relative to Earth

I will label the starting point "A" and the Island B. Hence 3 hrs later the "boat" is at point "C" 4 km South of B



$\therefore AC = \sqrt{16 + 32^2 - 2(4)(32)\cos 45}$
$= 29.3$ km.

$\therefore V_{B,E} = \dfrac{29.3}{3} = 9.77$ km/hr

using the Sine law

$$\frac{4}{\sin\theta} = \frac{29.3}{\sin 45} \Rightarrow \theta = 5.5^\circ$$

now we know the magnitude & direction of $\bar{V}_{BE}$ which will help us to determine $\vec{V}_{R,E}$

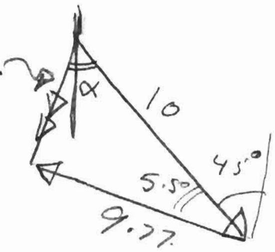
P.T.O

$\vec{V}_{RE} = \vec{V}_{R,B} + \vec{V}_{B,E}$ $\vec{V}_{RB}$ is given to be  10 km/hr.

using graphical methods.

$\therefore |\vec{V}_{RE}| =$

$= \sqrt{10^2 + 9.77^2 - 2(10)(9.77)\cos(5.5)}$



$|\vec{V}_{RE}| = 0.97$ km/hr.

using sine law $\frac{9.77}{\sin\alpha} = \frac{0.97}{\sin(5.5)}$

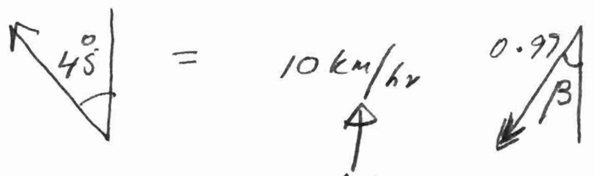
$\therefore \alpha = 74.87^\circ$

or we could say $\vec{V}_{RE} = 0.97$ km/hr 

where $\beta = 74.87 - 45 = 29.8^\circ$

now we can find $\vec{V}_{B,E}$ for any other value

$\vec{V}_{BE} = \vec{V}_{B,R} + \vec{V}_{R,E}$



need to find direction

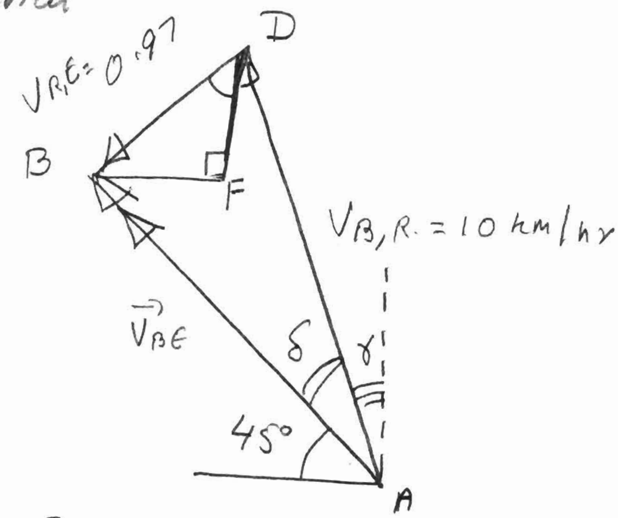
$\gamma + \delta = 45^\circ$

$B\hat{D}F = 29.8^\circ$

$\therefore D\hat{B}F = 60.2^\circ$

$\therefore A\hat{B}D = 105.2^\circ$

$\therefore$ using sine law



$\frac{10}{\sin(105.2)} = \frac{0.97}{\sin\delta}$

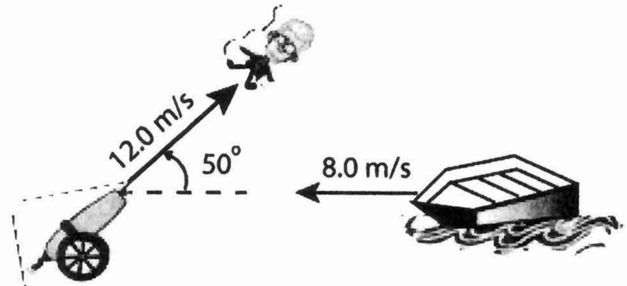
$\delta = 5.37^\circ$

$\therefore \gamma = 45 - 5.37 = \underline{\underline{39.6^\circ}}$

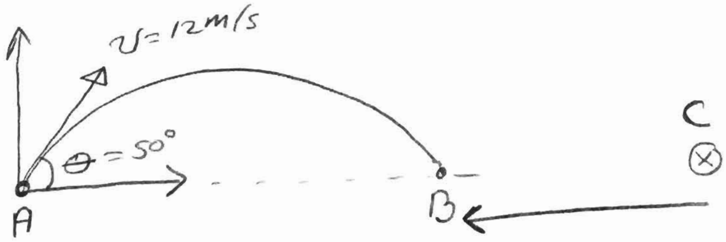
answer (6)

7) At the end of his term as Liberal leader, Stephane Dion was shot out of a cannon on the bank of the Ottawa river, at an angle of 50.0 degrees above the horizontal with a speed of 12.0 m/s. The horizontal component of Stephane's velocity is directed toward a barge that is approaching at a constant speed of 8.00 m/s, which will relay him safely back to his home riding in Montreal. What is the distance that the barge should be from the cannon when the disposed opposition leader is ejected? You can ignore air resistance and assume that Stephane lands on the barge at the same height from which he is shot from the cannon. Answer in meters.

(a) 26.3
(b) 29.5
(c) 37.2
(d) 39.6
(e) 42.1



While "Dion" has a horizontal displacement of AB The Barge moves from C to B.
For Dion in the horizontal direction



$(x-x_0) = v_{0x} t + \frac{1}{2} a_x t^2$

$(x-0) = 12 \cos 50\, t$ since $a_x = 0$

$(y-y_0) = v_{0y} t + \frac{1}{2} a_y t^2$

$(y-0) = (0-0) = 12 \sin 50\, t - 4.9 t^2$ since $a_y = 9.8\ m/s^2$

$\therefore t\ 1.88$ sec.

$\therefore (x-0) = 12\cos(50)(1.88) = 14.48$ m.

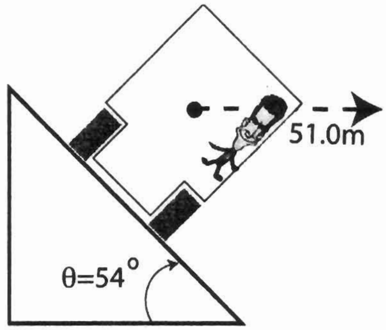
now $BC = 8t = 15.04$

$\therefore AC = AB + BC$
$= 14.48 + 15.04 = \underline{29.52\ m.}$

answer (b)

8) Jack Layton is ticked off at Michael Ignatieff for not voting down the latest federal budget, so he hands him over to someone to take him on a scary ride. Mr. Ignatieff is placed on the inside wall of a cargo truck as shown. The coefficient of static friction between the truck and Mr. Ignatieff is 0.86. Find the minimum speed the truck can have so Mr. Ignatieff is on the verge of sliding down the wall of the truck. The banking angle of the road is 54.0 degrees, and the radius of the curved road on which the truck drives is 51.0 m.

(a) 6.41 m/s
(b) 22.1 m/s
(c) 44.1 m/s
(d) 34.1 m/s
(e) 8.41 m/s



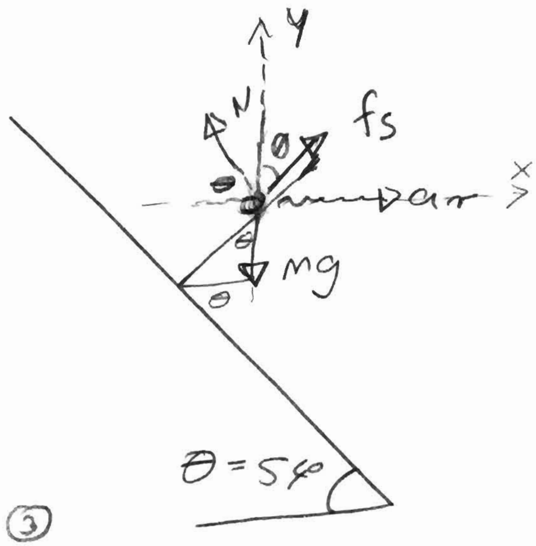
$\Sigma F_x = ma_x \quad \xrightarrow{+}$

$f_s \sin\theta - N\cos\theta = \frac{mv^2}{r}$ — (1)

$\Sigma F_y = ma_y \quad \uparrow$

$N\sin\theta + f_s\cos\theta - mg = 0$

$N\sin\theta + f_s\cos\theta = mg$ — (2)



$f_s = \mu_s N$

$\therefore \mu_s N\sin\theta - N\cos\theta = \frac{mv^2}{r}$ — (3)

$\mu_s N\cos\theta + N\sin\theta = mg$ — (4)

$\frac{(3)}{(4)} \quad \frac{\cancel{N}\mu_s\sin\theta - \cancel{N}\cos\theta}{\cancel{N}\mu_s\cos\theta + \cancel{N}\sin\theta} = \frac{v^2}{rg}$

$\therefore \left[\frac{\mu_s\tan\theta - 1}{\mu_s + \tan\theta}\right] rg = v^2$

$\left[\frac{(0.86)\tan 54 - 1}{0.86 + \tan 54}\right][51 \times 9.8] = v^2 = 41.05$

$\therefore v = 6.4$ answer (a).

9) A small mass m is tied to a ceiling of height $h=5\ell/4$ by a string of length ℓ and attached to the floor by a massless spring with spring constant $k=16mg/\ell$. When the mass is in its equilibrium position the spring is stretched by an amount $x=\ell/8$. The mass is pulled to the side until the spring has a length $s=3\ell/4$ and released from rest. What is the speed of the mass as it passes through its initial position (as shown in the left-hand figure)? (All units are in the MKS system)

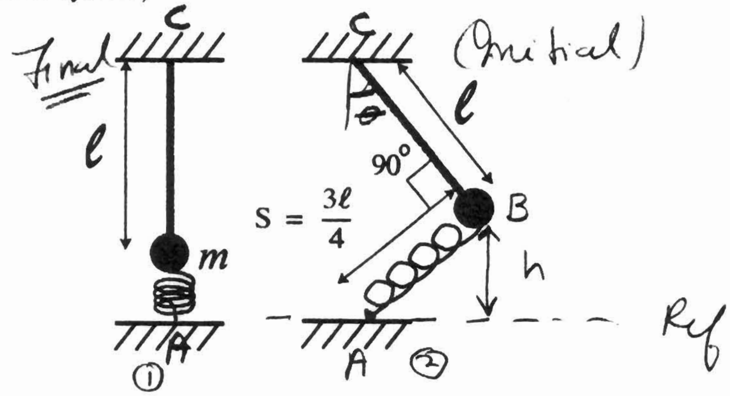
(a) $\sqrt{\frac{6g}{\ell}}$

(b) $\sqrt{32g\ell}$

(c) $\sqrt{\frac{32}{5}g\ell}$

(d) $\sqrt{6g\ell}$

(e) none of the above



after the spring has stretched [fig②] $AC = \frac{5\ell}{4}$

$\therefore \cos\theta = \ell/5\ell/4 = 4/5$ since there is no friction the mechanical energy is conserved.

$E_i = E_f \qquad E_i = (KE)_i + (U_g)_i + (U_s)_i$

U_g = gravitational potential Energy

U_s - spring potential Energy

$\therefore$ unstretched lenght of the spring $= \left[\frac{5\ell}{4} - \ell\right] - \ell/8$
$= \ell/8$

$E_i = 0 + mgh + \frac{1}{2}kx^2$
$= 0 + mg\left(\frac{5\ell}{4} - \ell\cos\theta\right) + \frac{1}{2}k\left(\frac{3\ell}{4} - \frac{\ell}{8}\right)^2 = \frac{9mg\ell}{20} + \frac{25mg\ell}{8}$

now the final Energy [fig①]

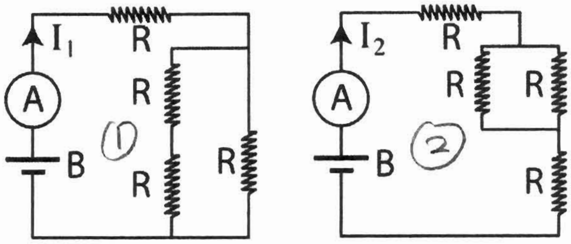
$E_f = \frac{1}{2}mv^2 + mg\left(\frac{\ell}{4}\right) + \frac{1}{2}k\left(\frac{\ell}{4} - \ell/8\right)^2$
$= \frac{1}{2}mv^2 + \frac{mg\ell}{4} + \frac{mg\ell}{8} = \frac{1}{2}mv^2 + \frac{3mg\ell}{8}$

Since $E_i = E_f \qquad \frac{9mg\ell}{20} + \frac{25mg\ell}{8} = \frac{1}{2}mv^2 + \frac{3mg\ell}{8}$

$v^2 = \frac{128\ell g}{20} = \frac{32}{5}\ell g$

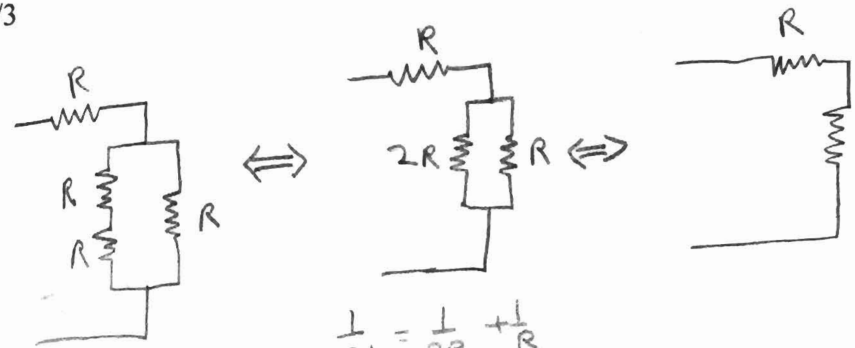
$\therefore v = \sqrt{\frac{32}{5}g\ell}$ answer ©

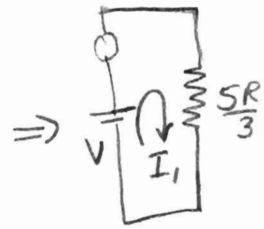
10) Four identical resistors, R, are connected to a battery B in the two circuits shown. Find the ratio of the currents I_1/I_2 measured on the ammeter A.



(a) 2/3
(b) 4/5
(c) 1
(d) 3/2
(e) 5/3

loop 1

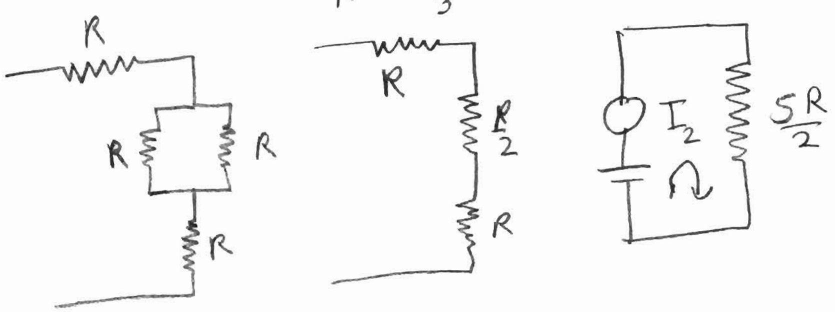




$$\frac{1}{R'} = \frac{1}{2R} + \frac{1}{R}$$
$$= \frac{1+2}{2R}$$
$$\frac{1}{R'} = \frac{3}{2R}$$
$$R' = \frac{2R}{3}$$

$$\therefore V = \frac{5R}{3} I_1 \quad \text{—(1)}$$

loop 2.



$$V = \frac{5R}{2} I_2 \quad \text{—(2)}$$

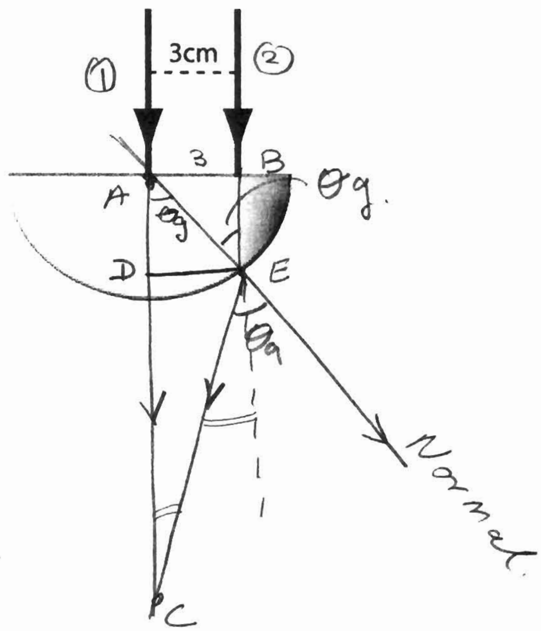
$$\frac{5R}{3} I_1 = \frac{5R}{2} I_2$$

$$\therefore \frac{I_1}{I_2} = \frac{3}{2}$$

answer (d)

11) Cheech and Chong shine two laser beams onto a semi-cylindrical glass block. Cheech shines his on the centre, while Chong's ray is 3.00 cm away. The two rays are parallel to each other and perpendicular to the flat surface of the block. The duo have decided to tour Ontario in their VW van, if the refracted rays intersect less than 20 cm from the flat surface. How far from the flat surface will the refracted rays intersect? Answer in cm.

$n_{glass} = 1.50$, $n_{air} = 1.00$,
Radius of block = 5.00 cm



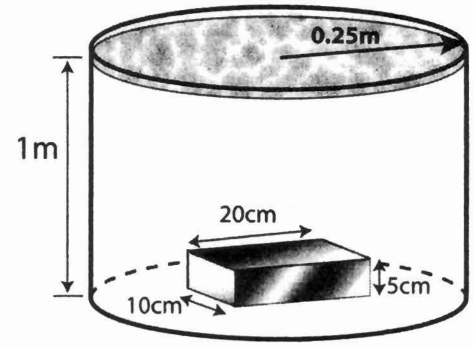
(a) 3.99
(b) 5.82
(c) 9.81
(d) 11.4
(e) Rays diverge away

Ray ① will pass through undeviated.
Ray ② will go through the flat surface and refract as shown away from the normal.
The 2 Rays will meet at point C.

Snell's Law
$$n_g \sin\theta_g = n_a \sin\theta_a$$
$$(1.5)\left(\frac{3}{5}\right) = 1 \sin\theta_a$$
$$\therefore \theta_a = 64.1^\circ$$
$$\theta_g = \sin^{-1}\left(\frac{3}{5}\right) = 36.9^\circ$$
$$\therefore D\hat{C}E = 64.1 - 36.9 = 27.2^\circ$$

need to find $AC = AD + DC$
from DEC triangle $\tan(27.2) = \frac{3}{DC}$
$$\therefore DC = 5.84$$
from ADE triangle $\tan\theta_g = \frac{3}{AD}$
$$\therefore AD = 3.99$$
$$\therefore AD + DC = 9.83 \text{ cm.}$$ answer (c).

12) Worried about future economic downturns, Isaac Newton hides a gold brick at the bottom of a large bucket filled with water. The brick has dimensions 20.0 cm x 10.0 cm x 5.00 cm, and the bucket is cylindrical with height 1.00 m and radius 0.250 m. If the density of water is 1.00 g/cm^3 and the density of the gold brick 19.3 g/cm^3, what is the total force acting on the gold brick?



(a) 9800 N
(b) 196 N
(c) 19.3 N
(d) 1.00 N
(e) none of the above

The block is not moving

$\therefore$ accelration $= 0$

$\Sigma F = ma = 0$

$\therefore$ net force is zero

answer (e)