2 Three class mates, Milly Ampere, Mike Rovolt and Meg Ohm are practicing their electric circuit skills in preparation for a test to be set by their teacher, Eddy Current. Milly connects 3 identical resistors, R, to a battery, B, as shown in the circuit below left, and Mike measures the current I_1 on ammeter A. Meg then connects the same 3 resistors as shown in the circuit below right, and Mike measures I_2. Find the ratio, I_2/I_1, of the ammeter readings.

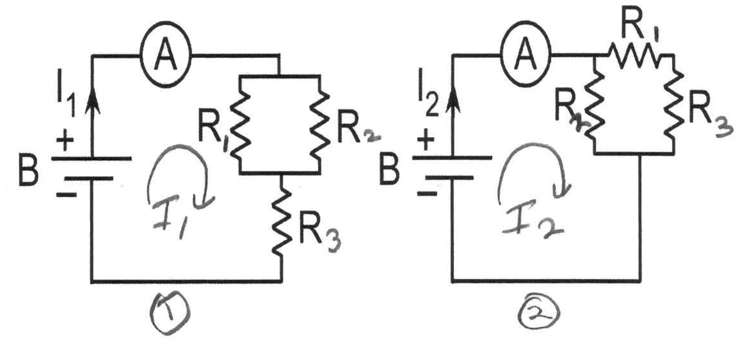


diagram ①

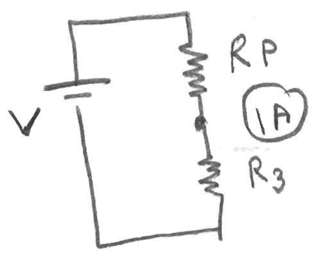
R_1 + R_2 are parallel

$\therefore \frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$

Since $R_1 = R_2 = R = R_3$

$\frac{1}{R_P} = \frac{2}{R} \Rightarrow R_P = R/2.$

diagram ① can be replaced by diagram (1A)



now R_P + R_3 are in Series.

$\therefore R_T = R_P + R_3 = R/2 + R$

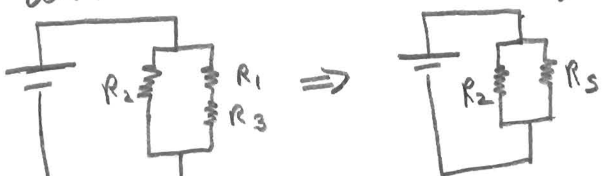
$\therefore R_T = \frac{3R}{2}$

$\therefore$ using ohm's law

$V = I_1 R_T$

$\boxed{\therefore I_1 = \frac{V}{R_T} = \frac{V}{3R/2} = \frac{2V}{3R}}$

now in diagram ② R_2 is parallel with the combination of R_1 + R_3



$R_S = R_1 + R_3 = 2R$ since they are in Series.

now the circuit changes to a single Resistor since R_2 is parallel to R_S

$\frac{1}{R_T} = \frac{1}{R_2} + \frac{1}{R_S} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}$

$\therefore R_T = \frac{2R}{3}$

using ohm's law

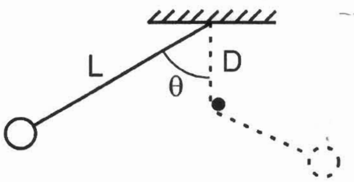
$\boxed{I_2 = \frac{V}{R_T} = \frac{3V}{2R}}$

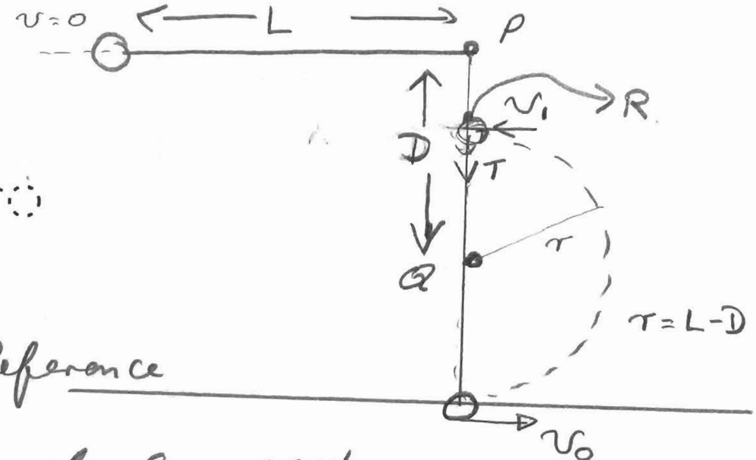
$\therefore \frac{I_2}{I_1} = \frac{3\cancel{V}}{2R} \cdot \frac{3\cancel{R}}{2\cancel{V}} = \frac{9}{4}.$

answer (D)

3 A simple pendulum consists of a small steel ball on the end of a string of length L, which can swing in the vertical plane. There is a peg located a distance D below the suspension point. The pendulum is released, from rest, from the horizontal position ($\theta = 90^{o}$). It swings down until the string hits the peg, after which the ball swings in a complete circle about the peg. Over which range of values of D shown below can this **never** happen?

(a) $3L/5 < D < L$
(b) $0 < D < 3L/5$
(c) $2L/3 < D < L$
(d) $0 < D < 2L/3$
(e) $0 < D < L$





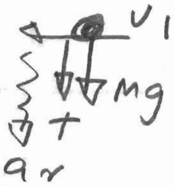
Since the object is released from rest using the conservation of Energy, we could find v_0, the speed at the reference (bottom) level.

$E_i = E_f$

$mgL = \frac{1}{2} m v_0^2 \quad \therefore v_0^2 = 2gL$ — ①

now the object will rotate about the peg "Q".

If the object makes it to the Top (position R) we could draw the f.b.d at R



$\sum \vec{F} = ma \quad \downarrow +ve.$

$mg + T = ma_r \Rightarrow mg + T = \frac{mv_1^2}{r}$ — ②

If it makes it to the to $T > 0$

$\therefore$ ② $\Rightarrow \frac{mv_1^2}{r} - mg > 0 \quad \therefore v_1^2 > rg \Rightarrow v_1^2 > (L-D)g$

so $\boxed{\frac{v_1^2}{L-D} > g}$ — ③. now using Conservation of Energy we can get another expression for v_1

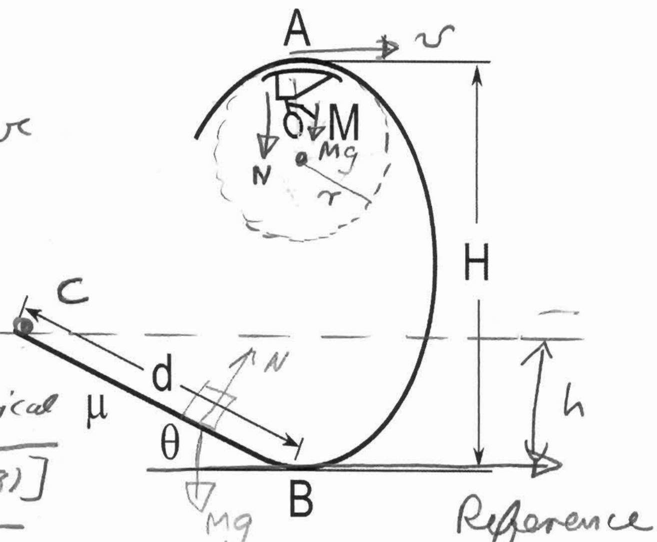
$E_i = E_f \Rightarrow \frac{1}{2} m v_0^2 = \frac{1}{2} m v_1^2 + mg(2r)$

$\therefore v_1^2 = v_0^2 - 2g(2r) = v_0^2 - 4g(L-D)$

— Continued —.

4 Al Gore rides in a bob sled coasting along a thrill-packed frictionless track built on the Columbia Ice Field. (Last chance before it melts!) As the sled, loaded with Mr Gore (total mass $M = 345$ kg), passes upside down through the peak of a loop at point A, the force exerted down on it by the track is 4560 N. Near point A the track follows a circular arc of radius 7.89 m. When the sled reaches point B, a distance $H = 43.2$ m lower than A, the track becomes a rough plane, inclined at $\theta = 33.3°$ above horizontal, with a coefficient of friction $\mu = 0.246$. Calculate the distance, d, the sled slides up the incline before first coming to rest. Answer in m.

(a) 45.2 (b) 64.3 (c) 32.1 (d) 58.8 (e) 69.5



at point "A" $\sum \vec{F} = Ma$ $\downarrow +ve$

$N + mg = ma_r = \dfrac{mv^2}{r}$

$\therefore v = \sqrt{\dfrac{r(N+mg)}{m}}$

we can now substitute the numerical values $v = \sqrt{\dfrac{(7.89)[4560 + (345)(9.8)]}{345}}$

≈ 13.5 m/s

We assume the sled comes to rest at "C".

$\therefore$ work done by friction from "B" to "C" will be $-f_k d$ and $f_k = \mu_k N = \mu_k mg \cos\theta$

$\therefore W_{friction} = -\mu_k Mg \cos\theta\, d = W_{nc}$

using the work Energy theorem.

$W_{nc} = \Delta KE + \Delta U_g = (0 - \frac{1}{2}Mv^2) + Mg(h - H)$

but $h = d \sin\theta$

$\therefore -\mu_k Mg(\cos\theta)d = -\frac{1}{2}\cancel{M}v^2 + \cancel{M}g\, d\sin\theta - MgH$

$\therefore d = \dfrac{\frac{1}{2}v^2 + gH}{g\sin\theta + \mu_k g\cos\theta}$ now sub in the numerial values

$= \dfrac{\frac{1}{2}(13.5)^2 + (9.8)(43.2)}{(9.8)\sin(33.3) + (9.8)(0.246)\cos 33.3} = \underline{\underline{69.5}}$

answer (E)

Since $v_0^2 = 2gL$

we can say $v_1^2 = 2gL - 4g(L-D)$

Hence using the condition developed in (3)

$$\frac{2gL - 4g(L-D)}{L-D} > g$$

$$4D - 2L \geqslant L - D$$

$$5D \geqslant 3L$$

$\therefore D \geqslant \frac{3}{5} L$ for the ball to swing in a complete circle

$\therefore$ The condition for it not to be able to swing should be

$$D < \frac{3}{5} L$$

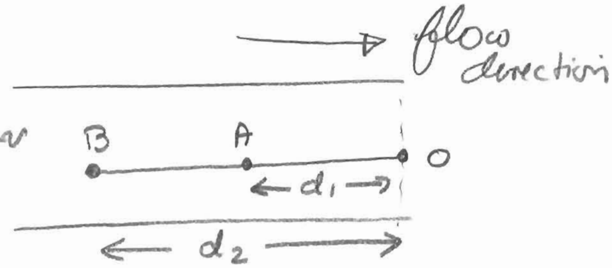
$$\therefore \quad 0 < D < \frac{3}{5} L$$

answer (B)

5 Little Willie, rowing his boat
Dropped his hat – but it stayed afloat;
When he noticed, he turned around,
Back at the start, his hat was found!

Willie starts to row from point O at time $t = 0$, moving upstream (against the current, which has a speed v). The boat's speed relative to still water is V. At time t_1, the hat falls overboard, but – not noticing – Willie continues to row upstream. At time t_2, Willie realizes he has lost his hat and quickly turns the boat around. At time t_3, Willie and the hat **simultaneously** reach the starting point, O. If the ratio V : v = 4 : 1, find the ratios of the times, $t_3 : t_2 : t_1$.

(a) 4 : 2 : 1 (b) 3 : 1.5 : 1 (c) 4 : 2.5 : 1 (d) 5 : 2 : 1 (e) 4 : 1.5 : 1



when moving upstream the speed of the boat relative to earth will be $V-v$
when moving down stream it will be $V+v$

$V = 4v$

$\therefore d_1 = (V-v)t_1$ and $d_2 = (V-v)t_2$

$\therefore t_1 = \dfrac{d_1}{V-v} = \dfrac{d_1}{3v}$ — ① and $t_2 = \dfrac{d_2}{3v}$ — ②

time taken for boat to come back to "O", from B $= t_3 - t_2 = \dfrac{d_2}{V+v}$ — ③

" " " " " " " A $= t_3 - t_1 = \dfrac{d_1}{v}$ — ④

" " " the hat " " " "

$\therefore$ ③ $\Rightarrow t_3 = \dfrac{d_2}{5v} + t_2 = \dfrac{d_2}{5v} + \dfrac{d_2}{3v} = \dfrac{8d_2}{15v}$

④ $\Rightarrow t_3 = \dfrac{d_1}{v} + t_1 = \dfrac{d_1}{v} + \dfrac{d_1}{3v} = \dfrac{4d_1}{3v}$

$\therefore \dfrac{\cancel{28}d_2}{\cancel{15}v} = \dfrac{4d_1}{3v} \Rightarrow 5d_1 = 2d_2 \Rightarrow \dfrac{d_2}{d_1} = 5/2 \Rightarrow d_2 = \frac{5}{2}d_1$

now from ① $\Rightarrow t_1 = \dfrac{d_1}{3v}$ — ⑤ from ② $\Rightarrow t_2 = \dfrac{5d_1}{6v}$ and $t_3 = \dfrac{4d_1}{3v}$

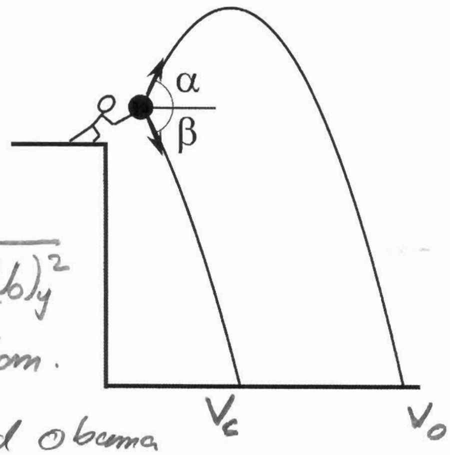
$\therefore t_3 : t_1 = \dfrac{4\cancel{d_1}}{\cancel{3v}} \times \dfrac{\cancel{3v}}{\cancel{d_1}} = 4$

$t_2 : t_1 = \dfrac{5\cancel{d_1}}{\cancel{6v}} \times \dfrac{\cancel{3v}}{\cancel{d_1}} = 5/2$

$\therefore t_3 : t_2 : t_1 = 4 : 5/2 : 1$ answer ©

6 Arnold Schwarzenegger throws Barack Obama and Hillary Clinton with equal speeds from the top of a cliff. Obama is thrown upwards at an angle α above the horizontal. Clinton is thrown downwards at an angle β below the horizontal. Which presidential candidate has the greater speed of impact with the water in the lagoon below? (Ignore air resistance.)

(a) Obama
(b) Clinton
(c) Both have the same speed of impact
(d) Depends on the candidates' masses
(e) Depends on α and β



$V_c = \sqrt{(V_c)_x^2 + (V_c)_y^2}$ and $V_o = \sqrt{(V_o)_x^2 + (V_o)_y^2}$

V_c & V_o are the speeds at the bottom.

The initial speed for clinton and obama are the same and lets call it v.

for Clinton.

$V_x = v\cos\beta \rightarrow$

$v_y = v\sin\beta \downarrow$

for Obama

$v_x = v\cos\alpha \rightarrow$

$v_y = v\sin\alpha \uparrow$

under the influence of gravity, we will not have a horizontal acceleration. $\therefore$ $(v_c)_x = v\cos\beta$, $(V_o)_x = v\cos\alpha$

using $v^2 - v_0^2 = 2a(s)$.

for clinton $\downarrow$ is +ve $\therefore a = g$

$\therefore (v_c)_y^2 = (v\sin\beta)^2 + 2gh$

$\therefore v_c = \sqrt{v^2\sin^2\beta + 2gh + v^2\cos^2\beta}$

$v_c = \sqrt{v^2(\sin^2\beta + \cos^2\beta) + 2gh}$.

$v_c = \sqrt{v^2 + 2gh}$.

for Obama $\downarrow$ +ve and $a = g$

$(v_o)_y^2 = (-v\sin\alpha)^2 + 2gh$

$\therefore v_o = \sqrt{v^2\sin^2\alpha + 2gh + v^2\cos^2\alpha}$

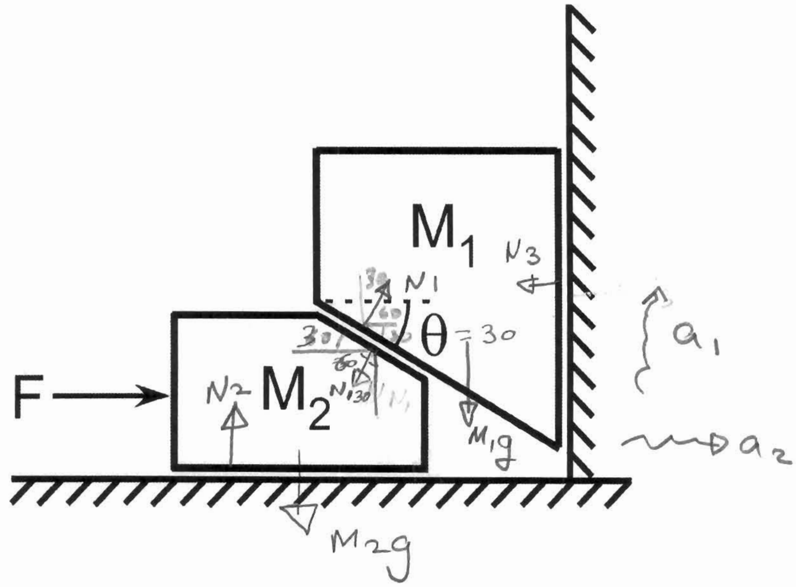
$= \sqrt{v^2(\sin^2\alpha + \cos^2\alpha) + 2gh}$

$v_o = \sqrt{v^2 + 2gh}$

$\therefore v_c = v_o$

7 Isaac Newton built the working model shown in the figure to test his ideas about motion. Depending on the values of the two masses, the horizontal applied pushing force, and the common slope angle, M_1 may slide up or down the vertical wall, and M_2 may slide in or out along the horizontal floor (assuming the sloping faces maintain sliding contact). All surfaces are frictionless. For $\theta = 30°$, $M_1 = 5$ kg, $M_2 = 3$ kg, and $F = 30$ N, calculate the magnitude of the initial acceleration of M_1. Answer in m/s^2.

(a) 0.21 (b) 0.53 (c) 1.0
(d) 0.94 (e) none of these



$\sum F_x = ma_x \longrightarrow +x$ for M_2

$F - N_1 \cos 60 = m_2 a_2$ — ①

$\sum F_y = ma_y$ ↑ +ve for M_1

$N_1 \cos 30 - M_1 g = M_1 a_1$ — ②

$\frac{a_1}{a_2} = \tan 30$ — ③

now we could rewrite ①

$\frac{F}{\cos 60} - N_1 = \frac{m_2 a_2}{\cos 60} \Rightarrow \frac{30}{\cos 60} - N_1 = \frac{3a_2}{\cos 60}$ — ④

Same for ② $\Rightarrow N_1 - \frac{m_1 g}{\cos 30} = \frac{m_1 a_1}{\cos 30} \Rightarrow N_1 - \frac{5g}{\cos 30} = \frac{5a_1}{\cos 30}$ — ⑤

④ + ⑤ $\frac{30}{\cos 60} - \frac{(5)(9.8)}{\cos 30} = \frac{3a_2}{\cos 60} + \frac{5a_1}{\cos 30}$ but $a_2 = \frac{a_1}{\tan 30}$

using this $\Rightarrow$ $60 - 56.58 = 6a_2 + 5.77a_1$

$3.42 = 10.39a_1 + 5.77a_1$

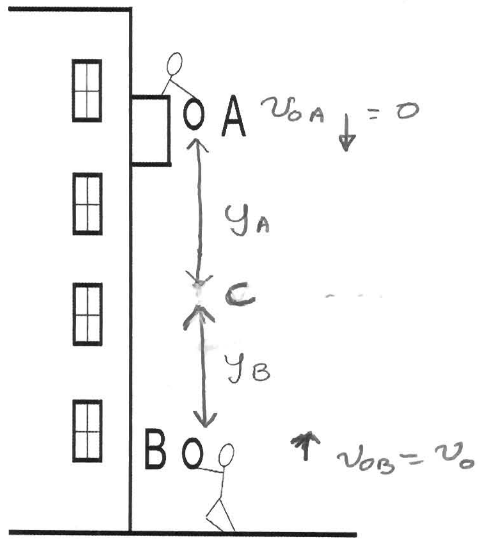
$3.42 = 16.16a_1$

$\therefore a_1 = \underline{\underline{0.21}}$ m/s^2.

answer ⓐ

8 Juliet stands on the balcony of her parents' high rise condominium. She drops a friendship ring, *A*, from rest down to Romeo, whose hand is 56.7 m directly below her dainty hand. At the same instant, Romeo expresses his friendship by throwing a ring, *B*, vertically upwards to Juliet. Unfortunately, the rings collide in midair. Just before they collide, the speed of *A* is exactly twice the speed of *B*. At what height (measured from Romeo's hand) do the rings collide? Answer in m.

(a) 22.7 (b) 29.5 (c) 37.8
(d) 43.8 (e) 45.0



$y_A + y_B = 56.7$ m.

let say the rings collide at point C

let the speed of ring A at C be v_A

and " " " " B " " " v_B

$\therefore$ using $v^2 - v_0^2 = 2a(y - y_0)$

we can say $v_A^2 = 2g\,y_A$ — ①

and $v_B^2 = -2g\,y_B + v_{0B}^2$ — ②

$\therefore$ ① − ② $\Rightarrow$ $v_A^2 - v_B^2 = 2g(y_A + y_B) - v_0^2$ — ③

However, $v_A = 2v_B$ $\therefore$ if we say $v_B = v$ then $v_A = 2v$

$\therefore$ ③ $\Rightarrow$ $4v^2 - v^2 = 2g(56.7) - v_0^2$

$3v^2 = 2(9.8)(56.7) - v_0^2$ — ④

We also know the time taken for A & B to reach C is the same.

$\therefore$ using $v = v_0 + at$ for A, $v_A = 0 + gt$ — ⑤

for B $v_B = v_0 - gt$ — ⑥

$\therefore$ ⑤ + ⑥ $\Rightarrow$ $v_A + v_B = v_0 = 3v$ now if we substitute this to ④

$3v^2 = 2(9.8)(56.7) - 9v^2$

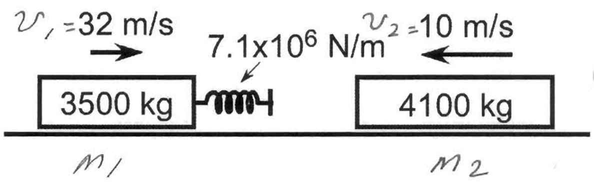
$\therefore 12v^2 = 1111.32$

$v = \pm 9.62$ m/s $\therefore$ $v_B = 9.62$ m/s, $v_0 = 28.86$ substitute into ②

$$y_B = \left[\frac{v_B^2 - v_{0B}^2}{-2g}\right] = \frac{(9.62)^2 - (28.86)^2}{-2(9.8)} = 37.77 \approx 37.8$$

answer (c)

9 Steven Harper and Stephane Dion, both in sleds, are on course for a collision, coasting directly towards each other on the frozen Rideau Canal in Ottawa. A compression spring of negligible mass is mounted on the front bumper of Dion's sled, as shown. Initial speeds, masses and spring constant are given in the figure. The collision is one dimensional, neither sled swerves, and the only force affecting the motion of the sleds (while in contact) is supplied by the spring. Find the maximum compression of the spring during the collision. Answer in m.



(a) 0.46 (b) 0.56 (c) 0.62 (d) 0.68 (e) 0.75

using conservation of momentum

$m_1\bar{v}_1 + m_2\bar{v}_2 = (m_1+m_2)\,v$

say $\rightarrow$ is +'ve. $\therefore\ m_1 v_1 - m_2 v_2 = (m_1+m_2)v$

$(3500)(32) - (4100)(10) = (7600)\,v \qquad \therefore\ v = +9.34.$

$\therefore$ The system is moving to the right, at maximum compression of the spring

using the conservation of energy

$E_i = E_f$

$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}kx^2$

$\therefore\ x = \sqrt{\dfrac{m_1 v_1^2 + m_2 v_2^2 - (m_1+m_2)v^2}{k}}$

$= \sqrt{\dfrac{(3500)(32)^2 + (4100)(10)^2 - (7600)(9.34)^2}{7.1\times 10^6}}$

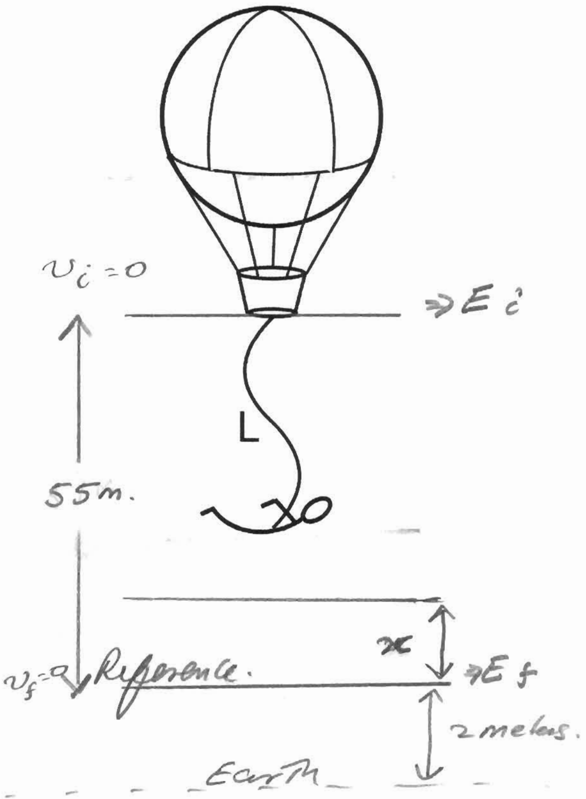
$= \underline{0.68}$ m. answer (d)

10 Belinda Stronach bungee jumps from the basket of a hot air balloon (at a fixed altitude of 57 m) and wishes, on her first bounce, to come within exactly 2.0 m of the ground, to strike a surprise blow on Tie Domi, who is standing directly below. In an earlier experiment she found that a 5.0 m length of bungee cord (manufactured by Magna) stretches by 1.5 m under her weight. What length, *L*, of bungee cord should she use? Answer in m.

(a) 42 (b) 62 (c) 120
(d) 68 (e) 26

If the length of the cord is "L" meters and extension is x meters

$x + L = 55$ — (1)



for a 5m cord the extension under her weight was 1.5m.

∴ for a 5m cord at equilibrium

$mg - kx = 0 \quad \therefore k = \frac{mg}{x} = \frac{mg}{1.5}$

now if we attach 2 identical springs in series

$\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2}$ where $k_1 = k_2$

$\therefore \frac{1}{K} = \frac{2}{k} \Rightarrow K = k/2$

if we has 3 identical springs in series

$\frac{1}{K} = \frac{3}{k}$ ∴ if we had n identical springs

$\frac{1}{K} = \frac{n}{k}$ now if the length of the cord was L meters.

then $\frac{L}{5} = n$ or we can say $L = 5n$

∴ the spring constant for the cord $= k/n = \frac{mg}{(1.5)n} = K$

$\therefore K = \frac{(5)(mg)}{(1.5)L}$

now using Conservation of Energy $mg(55) = \frac{1}{2}Kx^2$.

$mg(55) = \frac{1}{2}\left(\frac{5(mg)}{1.5L}\right)(55-L)^2 \Rightarrow 33L = 55^2 - 110L + L^2$

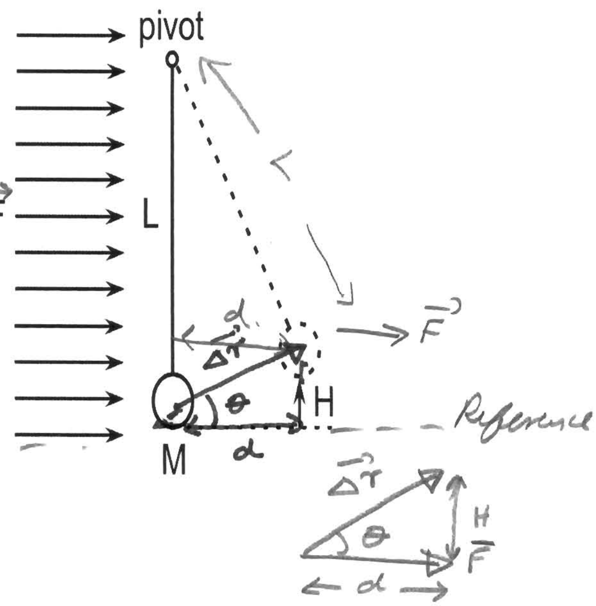
$\therefore L^2 - 143L + 3025 = 0 \quad \therefore L = \frac{143 \pm \sqrt{143^2 - 4(1)(3025)}}{2}$

$\therefore L = 117$ or $25.8 \approx 26$ m.

Clearly $L = 25.8$ m. ≈ 26 m. ∴ answer (e)

11 Kate Blanchett (mass $M = 62$ kg) is sitting on a swing with her centre of mass a distance $L = 1.7$ m from a fixed pivot point. She is “Blowing in the Wind”: A steady wind exerts on her a constant horizontal force $F = 340$ N. (Ignore the force of the wind on the ropes of the swing.) Initially Bob Dylan holds her in position so that the ropes of the swing are vertical. He then releases her from rest and she swings outward under the action of the force of the wind. She will first swing to a maximum height $H = H_{max}$, and then swing back down. Determine H_{max}, assuming the ropes of the swing always remain taut.
Answer in m.

(a) 0.81 (b) 1.2 (c) 0.22
(d) 0.87 (e) 2.6



Work $w = \vec{F} \cdot \vec{\Delta r}$

$= |\vec{F}||\vec{\Delta r}|\cos\theta$

Say $|\vec{\Delta r}|\cos\theta = d$

but $d = \sqrt{L^2 - (L-H)^2}$

$d = \sqrt{L^2 - L^2 + 2LH - H^2}$

$d = \sqrt{H(2L-H)}$

$\therefore w = Fd = F\sqrt{H(2L-H)}$

At this stage F becomes a non conservative force.

$\therefore$ Work done by a non conservative force $= W_{nc} = \Delta KE + \Delta U_g$

$\Delta KE = 0$ and $\Delta U_g = mgH$

$\therefore mgH = W_{nc} = F\sqrt{H(2L-H)}$

$\therefore m^2g^2H^{\cancel{2}} = F^2 \cancel{H}(2L-H)$

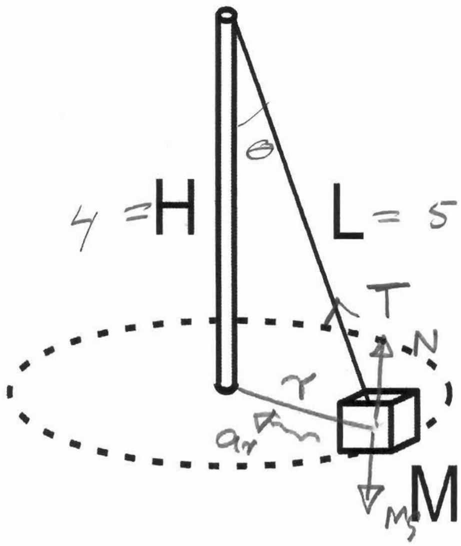
$\therefore H(m^2g + F^2) = F^2 2L$

$\therefore H = \frac{F^2 2L}{m^2g + F^2} = \frac{2L}{\frac{m^2g^2}{F^2} + 1} = \frac{3.4}{4.19} = 0.81\,m$

answer (A)

12 A mysterious visitor left a heavy sack filled with bank notes (total mass 45 kg) behind an Ottawa residence on Sussex Drive, hoping the occupant might find some use for it. He tied the sack to a light strong rope of length $L = 5.0$ m with the other end fastened to a pivot at the top of a vertical pole of height $H = 4.0$ m firmly anchored in the horizontal, frictionless ground. He then slid the sack outwards from the pole until the rope was taut, and then moved the sack at ever increasing speed along the ground following a circular path centered on the pole base. Calculate the maximum speed the block could be given if it just remained in contact with the ground, still travelling along the circle, when he slunk off into the night. Answer in m/s.

(a) 3.6 (b) 4.7 (c) 5.4
(d) 6.1 (e) 7.2



using $\sum \vec{F} = m\vec{a}$ in the radial direction

$$T\sin\theta = m a_r = \frac{mv^2}{r} \quad (1)$$

$$\sum \vec{F} = ma \uparrow$$

$$T\cos\theta + N - mg = 0 \quad (2)$$

we need $N > 0$

$$(2) \Rightarrow \therefore mg - T\cos\theta > 0 \quad \text{or} \quad mg > T\cos\theta$$

using (1) for $T \Rightarrow \cancel{m}g > \frac{\cancel{m}v^2}{r\sin\theta}\cos\theta$

$$\therefore v < \sqrt{\frac{rg\sin\theta}{\cos\theta}} \Rightarrow v < \sqrt{rg\tan\theta}$$

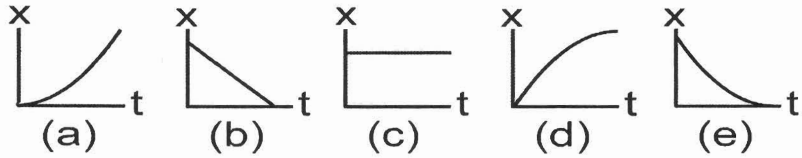
$$\tan\theta = 3/4 \qquad \therefore v < \sqrt{(3)(9.8)\tfrac{3}{4}}$$

$$v < 4.695$$

$$\Rightarrow v < 4.7 \text{ m/s}$$

answer (B)

1 Which of the following five displacement versus time graphs represents the motion of an object moving with a constant nonzero speed?



The most logical kinematics equation that describes the motion that has a relationship between displacement and time is

$(\bar{x}-\bar{x}_0) = \bar{v}_{0x} t + \frac{1}{2}\bar{a}_x t^2 \longrightarrow +ve$

since the object is said to be moving at a constant speed in a given direction, the acceleration should be zero. $\therefore a_x = 0$

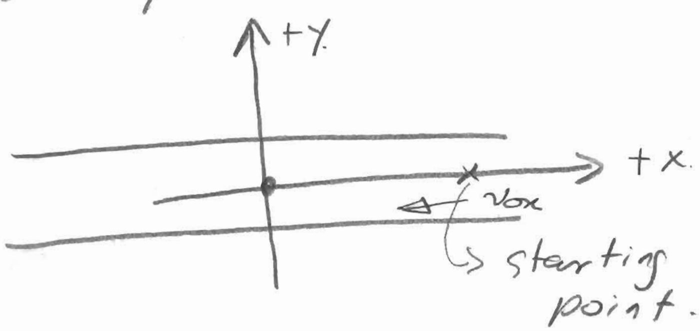
$\therefore \bar{x}-\bar{x}_0 = \bar{v}_{0x} t$

$\underbrace{\bar{x}}_{y} = \underbrace{v_{0x}}_{m} \underbrace{t}_{x} + \underbrace{x_0}_{b}$ Compare this to an equation of a straight line.

$\therefore$ it has to be either (b) or (c)

(c) indicates a zero slope. Hence (b) should be the answer. However, why do we see a negative slope. This could be, since the object may be moving in the negative "x" direction.

$\therefore v_{0x}$ is negative.



$\underbrace{x}_{y} = \underbrace{-v_{0x}}_{m} \underbrace{t}_{x} + \underbrace{x_0}_{b}$

(b)