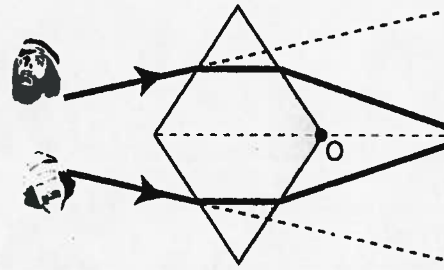
1) Cheech and Chong, finally united on Prancing with the Stars, take to the stage. As they dance, their sequined tiaras reflect the lasers used in the lighting production, and shine them through a sculptured ice block ready for the after-party, as shown. The ice block (n=1.47) is made of two equilateral triangular prisms of side length 50cm joined at their bases. The rays enter and emerge at points bisecting the sides of the triangles. How far from "O" will the laser beams meet? Answer in cm.

(a) 48.2
(b) 50.3
(c) 52.1
(d) 57.0
(e) 60.6



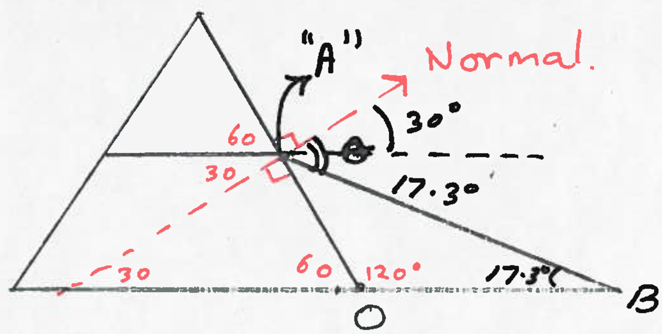
Snell's Law at pt "A"

$n_g \sin 30 = n_{air} \sin \theta$

$n_g = 1.47$, $n_{air} = 1$

$\therefore \theta = 47.3^\circ$

$\therefore A\hat{B}O = 17.3^\circ$



using Sine Law for the triangle OAB

$$\frac{OB}{\sin 42.7} = \frac{25}{\sin 17.3}$$ since $OA = 25$ cm

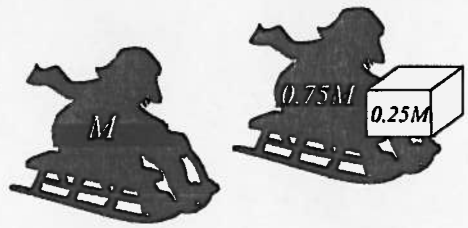
$\therefore OB = \underline{57.0 \text{ cm.}}$

2)Little Willie raced on the ice,
To beat his friend—that would be nice!
He'll try anything to be a winner,
Knows his Physics, this crafty SINner!

A.A.

Willie and his friend, Joe, race in identical massless sleds. Joe's mass is M, and since Willie is only $0.75M$, he agrees to carry a block of mass $0.25M$. Their teammates push Willie and Joe across the starting line ($x=0$) at the same time with equal speeds, v. The finish line is at $x=L$. When they pass the halfway point, Willie throws his block directly backwards at a speed v, relative to his moving sled. If Joe crosses the finish line at time T_1, and Willie at time T_2, find the ratio T_2/T_1. Assume the ice is frictionless.

(a) 3/4
(b) 4/5
(c) 5/6
(d) 6/7
(e) 7/8



Mass of Joe, $M_J = M$
Mass of Willie, $M_W = \frac{3}{4}M$
mass of Box (B), $M_B = \frac{1}{4}M$
Sled ⇒ S

$\vec{v}_{B,S} = \leftarrow v \qquad \vec{v}_{S,E} = \rightarrow v$

$\therefore \quad \vec{v}_{B,E} = \vec{v}_{B,S} + \vec{v}_{S,E} = 0$
$\qquad \leftarrow v + \rightarrow v$

$\Sigma \vec{p}_i = \Sigma \vec{p}_f \quad \rightarrow$

$(M_W + M_B)\,v = (M_B)(0) + M_W v'$

$Mv = \frac{3}{4}Mv' \Rightarrow v' = \frac{4}{3}v$

$\leftarrow L/2 \rightarrow . \leftarrow L/2 \rightarrow$

start finish

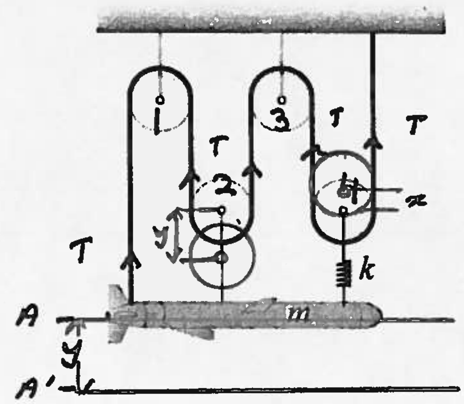
Time taken to reach the finish line

$T_2 = (L/2)/v + (L/2)/v' = \frac{L}{2v} + \frac{3L}{8v} = \frac{7L}{8v}$

$T_1 = L/v \qquad \therefore T_2/T_1 = \left(\frac{7L}{8v}\right)/L/v = \underline{\underline{7/8}}$

3)The Canadian and Russian navies square off for control of the North Pole. The Russians have high-tech nuclear submarines, while the Canadians have surplus submarines from the West Edmonton Mall. They use a system of massless pulleys, springs, and non-stretchable strings to load a torpedo filled with poutine into a tube, in hopes of giving their adversaries high cholesterol. The spring constant is k=3000N/m, and the mass of the torpedo is m=800kg. The torpedo is initially supported in such a way that the spring is not stretched, and then it is released from rest. What is the displacement of the torpedo from its initial position after the system reaches equilibrium? Assume the torpedo is horizontal in equilibrium. Answer in m.

(a) 0.418
(b) 0.786
(c) 1.05
(d) 1.42
(e) 2.09



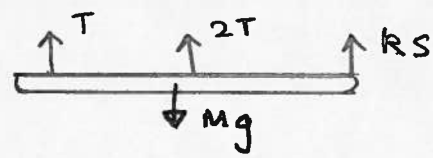
At Equilibrium Say
pulley ② drops "y" units
pulley ④ rises "x" units
Slack created will be $2x-2y$
The left most string hence must
extend by "y" units as well.

$$\therefore y = 2x - 2y \Rightarrow 3y = 2x \quad — ①$$

The spring extends $x+y$ units.

$\therefore$ extension of spring "s" $= x + y = 3y/2 + y = 5y/2$

now if we look at the forces



$2T = ks$

$\therefore \quad \Sigma F_y = 0 \uparrow \quad 5T - Mg = 0$

$\therefore T = Mg/5$

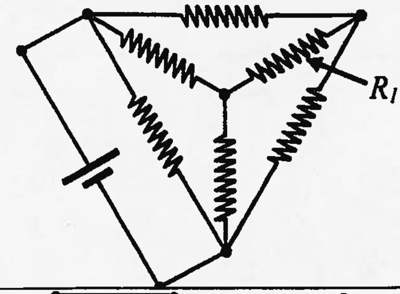
but $2T = ks \Rightarrow k5y/2 = 2(Mg)/5$

$$\therefore y = \frac{4mg}{25k} = \frac{(4)(800)(9.8)}{(25)(3000)}$$

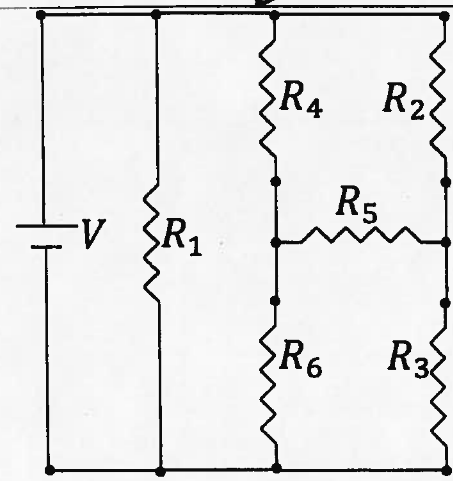
$$y = \underline{0.418 \text{ m.}}$$

4) The year is 1989, and Doc Brown takes Marty McFly forward in time to 2015. When Biff steals the DeLorean, Doc and Marty must make a new flux capacitor out of hoverboard parts and resistors, as shown. In order to properly connect the resistors, the duo must calculate the current through resistor R_I. If the battery has voltage V, and all resistors have resistance R, then what is this current?

(a) $V/3R$
(b) $V/2R$
(c) V/R
(d) $2V/R$
(e) None of the above



By Observation, if we redraw the diagram we notice that R_5 has equal potentials at each of its ends. Therefore, without a potential difference across it, R_5 cannot have a current flowing through it and $I_5 = 0.0$.



OR

$V_1 = V$

$V_4 = V_6 = \frac{V}{2}$

$V_2 = V_3 = \frac{V}{2}$

So $V_4 = V_2 = \frac{V}{2}$ and $V_5 = |V_6 - V_3| = \frac{V}{2} - \frac{V}{2} = 0$ $\therefore I_5 = \frac{V_5}{R_5} = \frac{0}{R} = 0.0$

Note: The "*sandbox*" results below are included in the ABCD choices.

$$I_1 = \frac{V_1}{R_1} = \frac{V}{R}$$

$$I_4 = I_6 = \frac{V_1}{R_4 + R_6} = \frac{V}{2R}$$

$$I_2 = I_3 = \frac{V_1}{R_2 + R_3} = \frac{V}{2R}$$

$$I_0 = I_1 + I_4 + I_2 = \frac{V}{R} + \frac{V}{2R} + \frac{V}{2R} = 2\frac{V}{R}$$

And

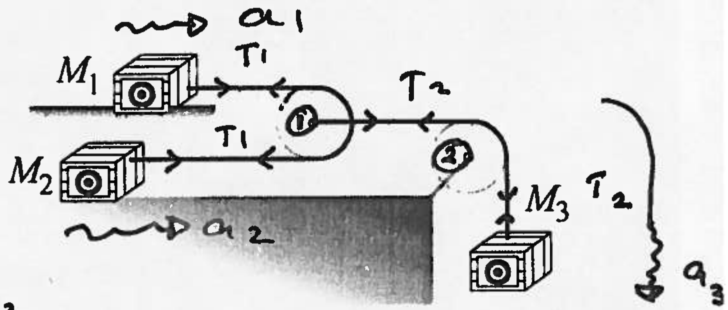
$$\frac{1}{R_{Tot}} = \frac{1}{R_1} + \frac{1}{R_4 + R_6} + \frac{1}{R_2 + R_3} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{2R} = \frac{2}{R}$$

$$\therefore R_{Tot} = \frac{R}{2}$$

And

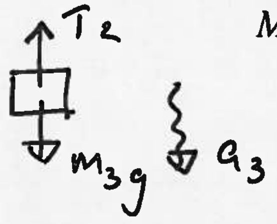
$$\therefore I_{Tot} = I_0 = \frac{V_{Tot}}{R_{Tot}} = \frac{V}{\frac{R}{2}} = 2\frac{V}{R}$$

5)Needing to offload unsold merchandise as they flee back to the USA, failed retailer Target devises a mechanism to dump boxes of junk off of the Niagara escarpment, one stop before the border. Managers attach three boxes, masses M_1=2kg, M_2=5kg, and M_3=7kg, to the pulley system as shown. Two masses sit on frictionless ice ledges, while the third dangles over the escarpment. The pulleys are frictionless and massless, and the strings are massless and non-stretchable. When the system is released from rest, the infallible business minds attempt to calculate the acceleration of M_1. What is it, in m/s^2?



(a) 7.71
(b) 8.21
(c) 9.23
(d) 10.3
(e) 11.5

for m_3



$\Sigma F_y = ma_y \downarrow$

$m_3 g - T_2 = m_3 a_3$ — ①

for pulley ① since it is massless $T_2 - 2T_1 = 0$

$\therefore \ T_2 = 2T_1$ — ②

for mass 1 $\Sigma F_x = ma_x \rightarrow$ $T_1 = m_1 a_1$ — ③

" " $T_1 = m_2 a_2$ — ④

and using conservation of string
you see that
$\bar{a}_1 + \bar{a}_2 = 2a_3$ — ⑤

now we have 5 equations with 5 unknowns.

③+④ $\Rightarrow m_1 a_1 = m_2 a_2$ also ②, ③+④ $\Rightarrow 2T_1 = m_1 a_1 + m_2 a_2 = T_2$

$\therefore \ T_2 = 2m_1 a_1$ — ⑥

sub into ① $\Rightarrow m_3 g = m_3 a_3 + 2m_1 a_1$

$\therefore \ m_3 g = m_3 \left[\frac{a_1}{2} + \frac{a_2}{2} \right] + 2m_1 a_1$ — ⑦

also since $m_1 a_1 = m_2 a_2 \Rightarrow a_2 = \frac{2}{5} a_1$

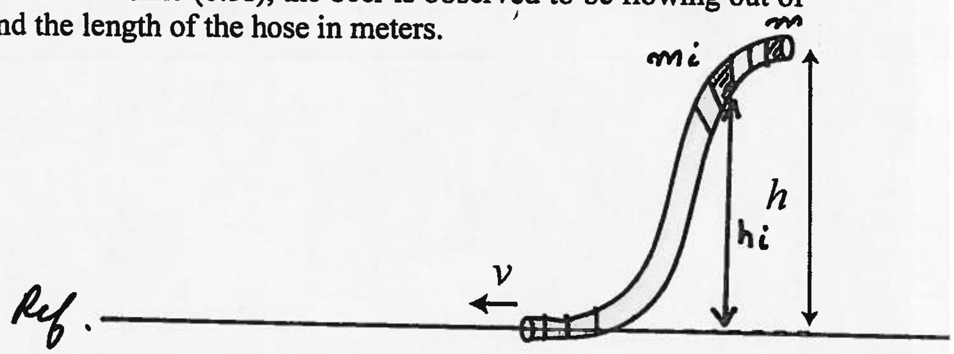
$\therefore \ m_3 g = \frac{m_3}{2} \left[a_1 + \frac{2}{5} a_1 \right] + 2ma_1$

$\therefore \ m_3 g = 8.9 a_1$ $\therefore \ a_1 = \frac{(7)(9.8)}{8.9} = 7.7079$

$\therefore \ a_1 \simeq \underline{\underline{7.71 \text{ m/s}^2}}$

6) It's Oktoberfest, and Bob and Doug McKenzie have filled a hose with non-alcoholic beer. Initially, the hose lies on a horizontal table with both ends plugged. The two hosers then raise one end of the hose to a height h=1.2m, while the other end remains on the table. Both ends are simultaneously unplugged, and after a short time (0.5s), the beer is observed to be flowing out of the hose with velocity v=3m/s. Find the length of the hose in meters.

(a) 0.980
(b) 1.96
(c) 2.94
(d) 3.92
(e) 4.89



Assume the Tube is divided into equal masses

$m, m_1, m_2, m_3 \cdots\cdots$

$m = m_1 = m_2 = m_3 = \cdots = m_i = M$

for a short time acceleration is a const.

$\therefore (x - x_0) = v_{0x} t + \frac{1}{2} a_x t^2$
$v = v_0 + a t$ $\Big\}$ $v_0 = v_{0x} = 0$

$\therefore \Delta x = \frac{1}{2} a t^2$
$v = a t$ $\Big\}$ $M = \rho A \Delta x$
where A = cross sectional area.

using conservation of Energy

$$(U_g)_i + (KE)_i = (U_g)_f + (KE)_f.$$

$$mgh + \cancel{m_1 g h_1} + \cancel{m_2 g h_2} + \cancel{m_i g h_i} \cdots + 0 = \cancel{m_1 g h_1} + \cancel{m_2 g h_2} + \cdots$$
$$\cdots \cancel{m_i g h_i} + \frac{1}{2}(M_{TOT}) v^2$$

$$mgh = \frac{1}{2}(m + m_1 + m_2 + \cdots) v^2$$

$$\rho A \Delta x\, g h = \frac{1}{2} \rho A l v^2$$ (l is the length of the tube)

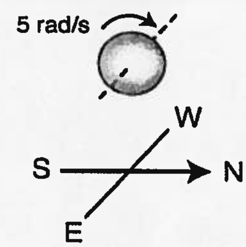
$$\therefore \frac{1}{2} a t^2 g h = \frac{1}{2} l v^2$$

$$a = v/t$$

$$\therefore \frac{1}{2} \frac{v}{t} t^2 g h = \frac{1}{2} l v^2$$

$$\therefore l = \frac{tgh}{v} = \frac{(0.5)(9.8)(1.2)}{3} = \underline{\underline{1.96\,m.}}$$

7) A hawk flies in the wind 30m above the flat prairies of Saskatchewan, momentarily stationary with respect to the air, which is moving 1m/s North. At that moment, the hawk sees a Blue Bombers fan, which surprises it so much that it lays a spherical egg. The egg pops out with zero velocity relative to the hawk, but spinning at 5 rad/s about an East-West axis parallel to the ground. If the egg is 10cm in diameter (which is negligible compared to 30m), what is the speed (relative to the ground) of the very top of the egg shell at the instant before the bottom of the egg hits the dirt? Answer in m/s.



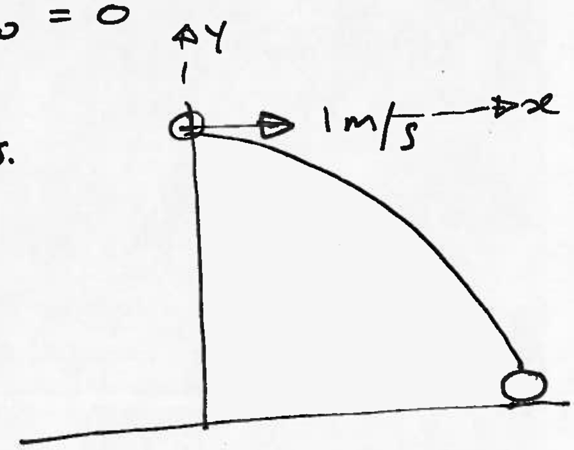
(a) 24.3
(b) 25.0
(c) 28.2
(d) 29.9
(e) 33.5

Velocity of The Hawk relative to Earth is 1 m/s $\rightarrow$

i.e $\vec{V}_{\omega,E} = \rightarrow 1\,m/s \qquad \vec{V}_{H,\omega} = \bar{0}$

$\therefore \vec{V}_{H,E} = \vec{V}_{H,\omega} + \vec{V}_{\omega,E}$

$= 0 + \rightarrow 1\,m/s.$



projectile motion

$v_y^2 - v_{oy}^2 = 2a_y(y - y_0)$

assume $\downarrow$ is positive.

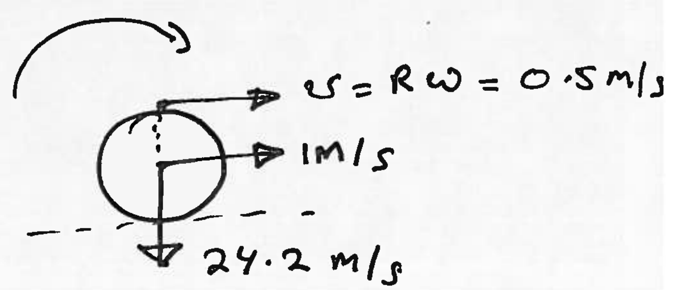
$\therefore v_y^2 - 0^2 = 2(9.8)(29.9)$

$v_y = 24.2$

$\therefore v = \sqrt{(24.2)^2 + (1.5)^2}$

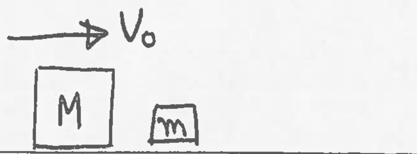
$= 24.25$

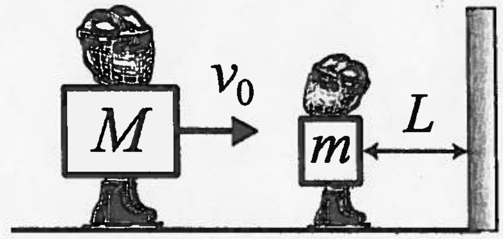
$\approx 24.3\ m/s.$



8)Zdeno Chara, mass M, is sliding with speed v_0 on frictionless ice towards Martin St. Louis, mass m. Martin stands defiantly at a distance L=2m from the boards, and Zdeno collides with him in a body check. Martin then slides towards the boards, bounces off, and collides with Zdeno again. Martin proceeds to bounce back and forth between Zdeno and the boards at an ever-increasing rate, as Zdeno continues to move to the right. Making the approximation $M>>m$ (say M=100m), and assuming that all collisions are elastic, approximately how close does Zdeno come to the boards before Martin stops him and begins to repel him? Answer in meters.

(a) 1/10
(b) 1/5
(c) 3/10
(d) 2/5
(e) 1/2





1st Collision. $\Sigma \vec{p}_i = \Sigma \vec{p}_f \rightarrow$

$$MV_0 = MV_1 + mv_1 \quad \text{— (1)}$$

$(KE)_i = (KE)_f$ since it is elastic

$$\tfrac{1}{2}MV_0^2 = \tfrac{1}{2}MV_1^2 + \tfrac{1}{2}mv_1^2$$

$$M(V_0^2 - V_1^2) = mv_1^2 \Rightarrow M(V_0+V_1)(V_0-V_1) = mv_1^2 \quad \text{— (2)}$$

$$(1) \Rightarrow M(V_0 - V_1) = mv_1 \qquad \therefore \frac{(2)}{(1)} \Rightarrow \boxed{V_0 + V_1 = v_1}$$

$$\therefore MV_0 - MV_1 = m(V_0 + V_1) \qquad \therefore \text{ If } M >> m$$

$$\therefore (M-m)V_0 = V_1(M+m)$$

$$V_1 \simeq V_0 \quad \text{and} \quad v_1 = 2V_0$$

Since these are elastic collisions

1st collision $\quad \tfrac{1}{2}MV_0^2 = \tfrac{1}{2}MV_1^2 + \tfrac{1}{2}mv_1^2$

2nd $\quad$ " $\quad \tfrac{1}{2}MV_1^2 + \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}MV_2^2 + \tfrac{1}{2}mv_2^2$

$\vdots$

final collision $\quad \tfrac{1}{2}MV_{n-1}^2 + \tfrac{1}{2}mv_{n-1}^2 = \tfrac{1}{2}MV_n^2 + \tfrac{1}{2}mv_n^2$

if "n" is the final collinsion $V_n = 0$ $v_n = v_f$.

Hence the transfer of KE $\Rightarrow$

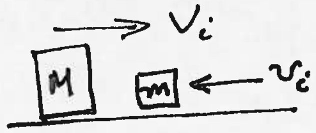
$$\tfrac{1}{2}MV_0^2 = \tfrac{1}{2}mv_f^2$$

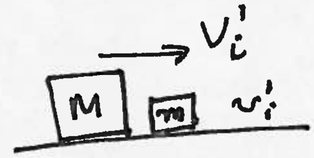
$$v_f = \sqrt{\frac{M}{m}}\, V_0$$

(next)

(8) continued.

just before
and after a collision





$$\Sigma \bar{p}_i = \Sigma \bar{p}_f \longrightarrow MV_i - mv_i = MV_i' + mv_i'$$

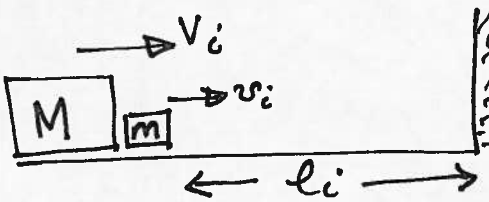
$$M(V_i - V_i') = m(v_i' + v_i) \quad \text{—} \ (3)$$

$$(KE)_i = (KE)_f \Rightarrow \tfrac{1}{2} MV_i^2 + \tfrac{1}{2} m v_i^2 = \tfrac{1}{2} MV_i'^2 + \tfrac{1}{2} m v_i'^2$$

$$\therefore M(V_i + V_i')(V_i - V_i') = m(v_i' + v_i)(v_i' - v_i) \quad \text{—} \ (4)$$

$$\frac{(4)}{(3)} \Rightarrow V_i + V_i' = v_i' - v_i \Longrightarrow \boxed{V_i + v_i = v_i' - V_i'} \quad \text{—} \ (5)$$

The distance remaining to the wall just before the next collision



Say it takes time "t" for the next collision.

then $V_i t = 2l_i - v_i t$

or $V_i t + v_i t = 2 l_i \quad \therefore t = \dfrac{2 l_i}{V_i + v_i}$

The distance "M" moves hence will be

$$l_i - l_i' = V_i t \Rightarrow l_i' = l_i - V_i t$$

$$\therefore l_i' = l_i - \frac{V_i \, 2 l_i}{V_i + v_i} \Longrightarrow l_i'(v_i + V_i) = l_i (v_i - V_i)$$

using (5) $\Rightarrow l_i'(v_i' - V_i') = l_i(v_i - V_i) \Rightarrow (6)$

using (6) if l_i' = final l_f and l_i = initial L

$$l_f(v_f - V_f) = L(v_1 - V_1) \Rightarrow V_f = 0, \ v_f = \sqrt{\frac{M}{m}} V_0$$

$$V_1 = V_0 \quad v_1 = 2V_0$$

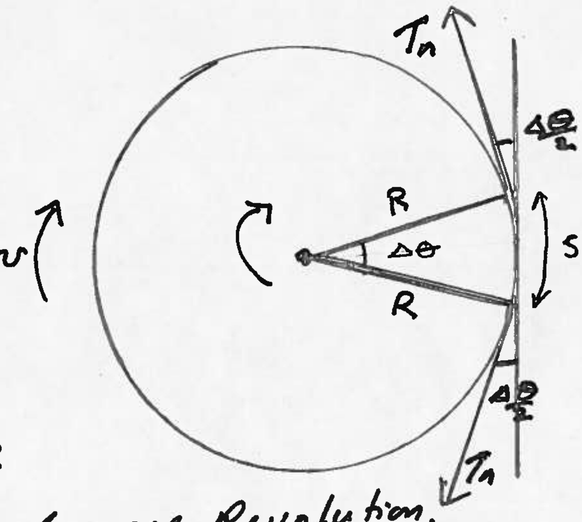
$$\therefore L(2V_0 - V_0) = l_f\left(\sqrt{\frac{M}{m}} V_0 - 0\right)$$

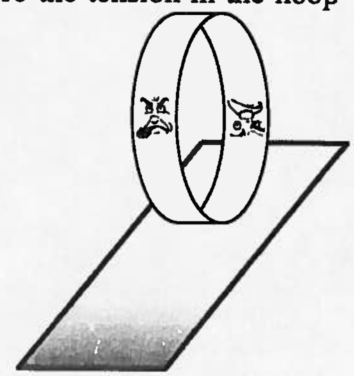
$$\therefore l_f = L\sqrt{\frac{M}{m}} = \frac{L}{10} \quad \text{if } L = 2$$

$$\underline{l_f = \tfrac{1}{5} \ m}$$

9) During a carbon tax skirmish, the doors of Parliament burst open and Stephen Harper and Justin Trudeau roll out in a "human hamster wheel," hands grabbing each other's ankles. In this hoop-like configuration, the duo begins to rotate faster as they roll down the building's wheelchair ramp, thus increasing the tension in their "hoop." Approximate the mass of the dignitaries as uniformly distributed around a thin hoop of radius 1m, and assume the maximum force of each man's grip is equal to the weight of both men combined. What is the maximum rotation rate (revolutions per second) that the men can withstand before the tension in the hoop exceeds their grip?

(a) 0.884
(b) 1.25
(c) 1.56
(d) 1.77
(e) 7.85





$v = \frac{2\pi R}{T}$

T = Time for one Revolution.

mass of The small segment $\Delta m = \frac{M}{2\pi}\Delta\theta$

centripital acceleration $\Rightarrow a_r = \frac{v^2}{R} = \left(\frac{4\pi^2 R^2}{T^2}\right)/R$

$\therefore$ The Radial force $F_r = \Delta m\, a_r = \frac{\Delta m\, 4\pi^2 R}{T^2}$

$\therefore F_r = \frac{M}{2\pi}\frac{\Delta\theta\, 4\pi^2 R}{T^2} = \frac{M(\Delta\theta)\, 2\pi R}{T^2}$

using The Tension "T_n" $F_r = 2T_n \sin\frac{\Delta\theta}{2}$

for small angles $\sin\alpha \approx \alpha$ $\therefore F_r = 2T_n\frac{\Delta\theta}{2} = T_n\Delta\theta$

$\therefore T_n\Delta\theta = \frac{M(\Delta\theta)\, 2\pi R}{T^2}$

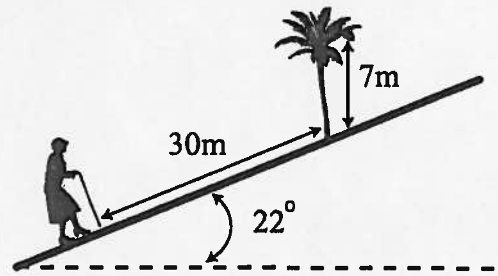
for a maximum $T_n = Mg$

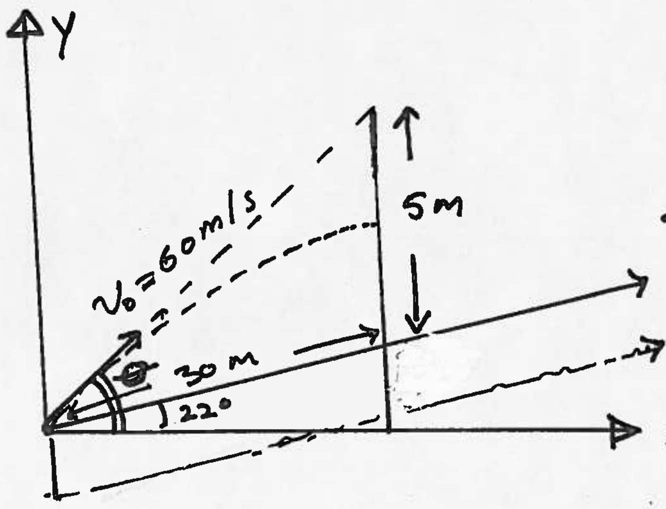
$\therefore Mg = \frac{M\, 2\pi R}{T^2}$ $\therefore T = \sqrt{\frac{2\pi R}{g}}$ $\frac{1}{T} = f$

$\therefore f = \sqrt{\frac{g}{2\pi R}} = 1.25$ rev/sec.

10)A snowbird is stranded in Florida after she misses the last flight back to Canada in spring. In order to eke out a living over the long USA summer, she decides to harvest and sell coconuts. She is standing on a hill sloped at 22 degrees, with her feet 30m from the base of a coconut tree, which has a coconut 7m above its base. She holds a stone aimed directly at the coconut, waiting for it to fall. At the instant she sees it fall, she throws the stone with a speed of 60 m/s (she is a retired superstar pitcher!). Unfortunately, she has neglected to take into account her reaction time of 0.2s. If she releases the stone 2m above her feet, how far directly above the coconut does the stone pass? Answer in m.

(a) 0.562
(b) 1.05
(c) 1.25
(d) 1.41
(e) 5.59





$X = 30 \cos 22 = 27.82$

$Y = 5 + 30 \sin 22 = 16.2$

$\tan \theta = Y/x$

$\therefore \theta = 30.28°$

$v_0 = 60 m/s.$ $\quad v_{ox} = 60 \cos 30.28 = 51.82$ m/s

$v_{oy} = 60 \sin 30.28 = 30.08$ m/s

time taken to travel x $\Rightarrow (x - x_0) = v_{ox} t \rightarrow$

$\therefore t = 0.54$ sec.

during time the Vertical Motion

$(y - y_0) = v_{oy} t + \frac{1}{2} a_y t^2 \uparrow \Rightarrow (30.08)(0.54) - (4.9)(0.54)^2$

$Y_1 = 14.82$ m.

During this time plus reaction

$y - 16 = 0 - (4.9)(.54 + .2)^2.$

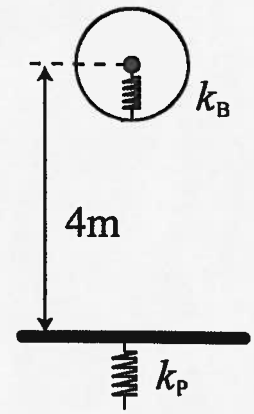
$y = 13.58$ m.

$\therefore Y_1 - y = 1.24$ m

$\simeq 1.25$ m.

11) The Canadian cricket team is designing a new cricket ball in an attempt to gain an advantage against the mighty Sri Lankans in the Waterloo World Cup (WWC). The ball can be modeled as a point mass of 3kg connected to a massless sphere of radius 14cm by a massless spring with spring constant k_B=600N/m, as shown. The Canadi-ans test the ball by dropping it from rest, with its centre 4m above a springy non-regulation pitch. The pitch is modeled as a massless platform connected to a massless spring with spring constant k_P=240N/m. Neglecting air resistance, find the maximum displacement of the pitch (in meters) during the test.

(a) 0.436
(b) 0.578
(c) 0.684
(d) 0.731
(e) 0.954



Let x_B and x_p be the compressions of the springs

$\therefore$ Conservation energy.

$$mg\left[4 - .14 + x_B + x_p\right] = \frac{1}{2} k_B x_B^2 + \frac{1}{2} k_p x_p^2.$$

$$k_B x_B = k_p x_p \quad \therefore x_B = \frac{k_p}{k_B} x_p$$

$$\therefore mg\left[4 - .14 + x_p\left[\frac{k_p}{k_B} + 1\right]\right] = \frac{1}{2} x_p^2 \left[k_B \frac{k_p^2}{k_B^2} + k_p\right]$$

Sub in the numerical values

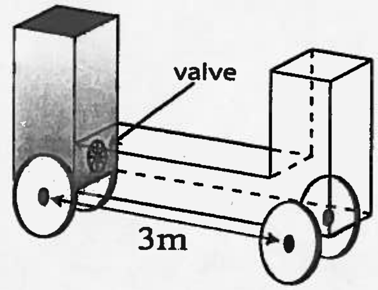
$$113.59 + 41.2\, x_p = 168\, x_p^2$$

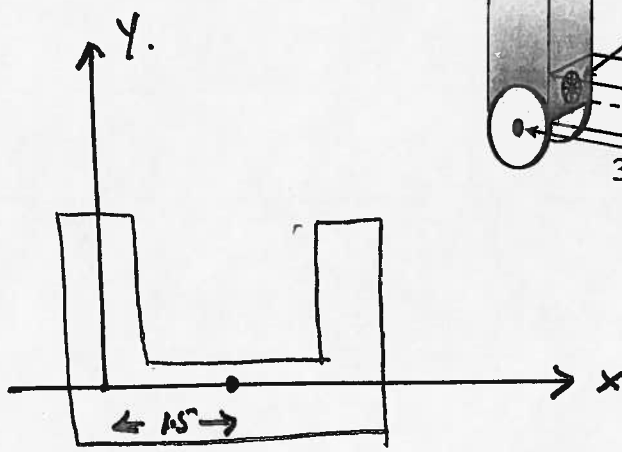
Solve the quadratic $\quad x_p = \dfrac{41.2 \pm 279.35}{336}$

$$= 0.954 \text{ m.}$$

12)Seeking fame and fortune on Dragon's Den, Dr. Doofus has constructed a race car consisting of a U-shaped tube of mass M (including him), attached to massless, frictionless wheels with wheelbase 3m, whose axles pass through a point directly below the centers of mass of the vertical arms of the U-tube. He has filled the front arm of the U-tube with a secret patented hyper-dense liquid (called Doofusium) of total mass $3M$, which is stopped from flowing into the rest of the U-tube by a massless valve. Starting from rest in front of the skeptical Dragons, Doofus opens the valve. How far will his U-tube race car travel by the time the liquid reaches equilibrium again? Answer in m.

(a) 3/8
(b) 3/4
(c) 9/8
(d) 3/2
(e) 9/4





In The absence of any net external force.

$$(X_{cm})_i = (X_{cm})_f.$$

$$\frac{\sum m_i x_i}{\sum m_i} = \frac{\sum m_f x_f}{\sum m_f}.$$

$$\frac{3M(0) + M(1.5)}{4M} = \frac{(1.5 - d)\,4M}{4M}$$

$$\therefore \frac{1.5}{4} = 1.5 - d.$$

$$\therefore d = 1.125 = 9/8 \text{ m.}$$