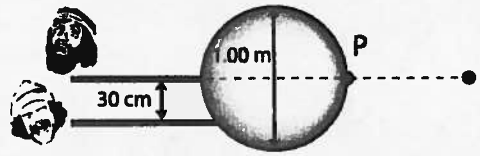
2014 - 46th SIN Test

1) Cheech and Chong lay on the couch, trying to get their pet goldfish to chase laser pointers. Cheech notices that, when he shines his laser pointer through the centre of the spherical fishbowl, he can see a spot directly behind it on the wall. Chong shines his beam parallel to Cheech, but 30cm below. He notices that his spot on the wall falls directly on top of Cheech's. If the fishbowl has a diameter of 1.00m, calculate the distance from point P to the spot on the wall. (Answer in cm.)

(a) 17.0
(b) 17.8
(c) 37.7
(d) 46.2
(e) 67.0



We assume it is a thin wall fish bowl

we must remember that $n_{water} = 1.33$

R = Radius = 50 cm

$\therefore$ drawing the ray diagram we see

that $\theta_1 = \tan^{-1}(30/40) = 36.87°$

using Snell's law at A

$\Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_2 = 26.81°$

$\alpha = 90 - \theta_1 = 53.13°$, $\beta = 10.06°$

$\therefore \gamma = 126.38°$

now we could get $C\hat{O}D = 16.75° = \delta$

$\sin\delta = \frac{x}{50} = \frac{CD}{OC} \Rightarrow CD = x = 14.4\,cm$

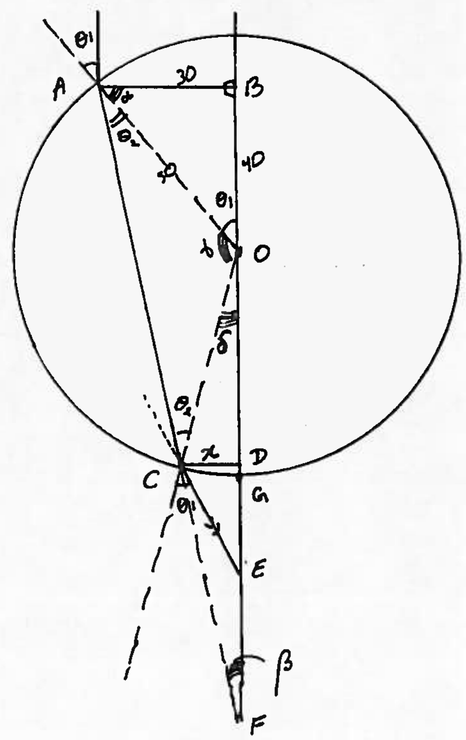
$\therefore C\hat{E}D = [(\theta_1 - \theta_2) + \beta] = 20.66°$

now $\tan C\hat{E}D = \frac{CD}{DE} \Rightarrow DE = 38.19\,cm$

$DG = 50 - 50\cos(C\hat{O}D) = 0.80$

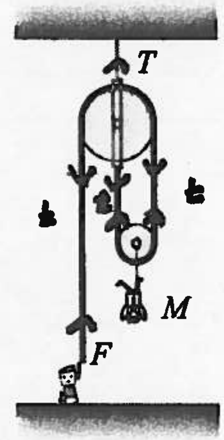
$\therefore GE = DE - DG = 37.38\,cm \approx 37.7\,cm$

$\therefore$ answer is (c)



2) Little Willie played a trick,
On his friend, whose name was Nick.
Hooked his belt and raised him high,
Laughed so hard, thought he would die!

A.A.



Willie uses the pulley system as shown to raise Nick. He then exerts a force, F, such as to hold Nick (mass M) with the ropes and pulleys all stationary. Assume that the pulleys are massless and frictionless. Find the ratio $R = T/F$, where T is the tension in the cable holding up the large pulley.

(a) 0.50 **(b)** 1.00 **(c)** 1.50 **(d)** 2.00 **(e)** 3.00

F.B.D for mass $\quad \uparrow 2t \quad \downarrow Mg$

$\Sigma F = ma \uparrow \quad 2t - Mg = 0 \Rightarrow t = Mg/2$ — ①

F.B.D for large pulley $\quad \uparrow T \quad \downarrow 3t$

$\Sigma F = ma \uparrow \quad T - 3t = 0 \Rightarrow T = 3t$ — ②

F.B.D for hand pulleys $\quad \uparrow t \quad \downarrow F$

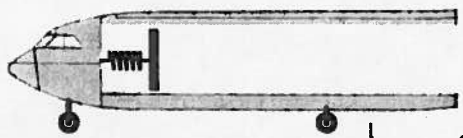
$\Sigma \bar{F} = m\bar{a} \uparrow \Rightarrow t - F = 0 \Rightarrow t = F$ — ③

$\therefore \frac{②}{③} \Rightarrow \frac{3t}{t} = \frac{T}{F} = 3$

answer (e)

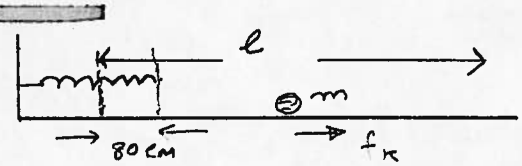
3) P. M. Harper (mass 82kg) has an ultra-light airplane (mass 400kg) sitting at rest on the tarmac. The plane is free to roll on the ground without friction. He boards the plane in true Prime Ministerial style by jumping from the roof of his moving limo into the cargo door at the tail of the plane. When he lands, he is moving at 8m/s horizontally towards the front of the plane. To his surprise, he slides the whole length of the aisle to the pilot's door. In anticipation of Harper's botched boarding, the pilot has equipped the door with cushions, equivalent to a spring, with spring constant 3000N/m. If the maximum compression of the spring is 80cm, and the coefficient of friction between Harper's ice-covered boots and the floor is 0.09, how far does Harper slide along the aisle? (Answer in m.)

(a) 0.00
(b) 15.9
(c) 16.8
(d) 23.0
(e) 43.3



$m = 82\text{ kg}$ $\quad v = 8\text{ m/s}$

$M = 400\text{ kg}$



Conservation of linear momentum

$$mv = (M+m)V \Rightarrow V = \frac{mv}{M+m}$$ (now the spring is fully compressed)

Work Energy Theorem (one version).

$$W_{nc} = \Delta KE + \Delta U_g + \Delta U_s$$

W_{nc} = work done by non conservative forces.

$$-\mu_k m g l = \frac{1}{2}m(V^2 - v^2) + \frac{1}{2}MV^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}mv^2\left[\frac{m^2}{(M+m)^2} - 1\right] + \frac{1}{2}M\left(\frac{m^2v^2}{(M+m)^2}\right) + \frac{1}{2}kx^2$$

$$= \frac{1}{2}mv^2\left[-0.97\right] + \frac{1}{2}(400)v^2\left[0.029\right] + \frac{1}{2}(3000)(.8)^2$$

$$-\mu_k m g l = -1218.25$$

$$\therefore l = \frac{1218.25}{mg\mu_k} = 16.84\text{ m}$$

answer (c)

4) Two physics students, Alice and Bob, living in neighbouring college rooms, decide to economize by connecting their ceiling lights in series. They agree that each will install a 100W bulb in their own rooms and that they will pay equal shares of the electricity bill. However, both decide to try to get the better lighting at the other's expense: Alice installed a 200W bulb and Bob installed a 50W bulb. Which student has the brighter lighting?

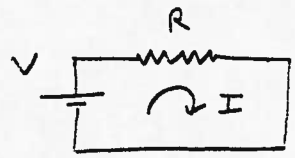
(a) Alice
(b) Bob
(c) Both have the same brightness
(d) Alice's bulb instantly burns out
(e) Answer cannot be determined

Bob $\Rightarrow$ 50W

Alice $\Rightarrow$ 200 W

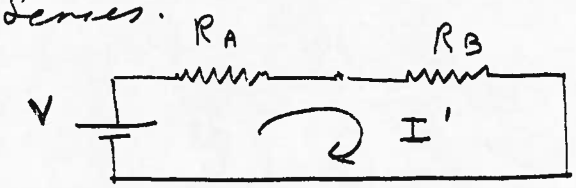
If we look at one bulb at a time

ohm's Law $\Rightarrow$ $R = \frac{V}{I}$ $\quad P = \frac{V^2}{R}$



$$\frac{P_B}{P_A} = \frac{R_A}{R_B} = \frac{50}{200} = \frac{1}{4} \quad \text{---} \ 1$$

Now when they are in Series.



$$P = (I')^2 R$$

$$\frac{P'_B}{P'_A} = \frac{\text{New Power of Bob}}{\text{New Power of Alice}}$$

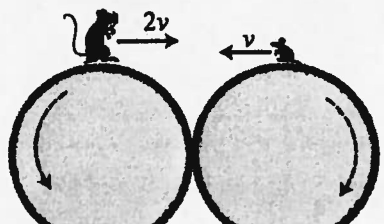
$$= \frac{(I')^2 R_B}{(I')^2 R_A} = \frac{R_B}{R_A} = 4$$

$\therefore$ $P'_B = 4 P'_A$

$\therefore$ $P'_B > P'_A$ $\quad \therefore$ answer is b

5) During a log drive down the mighty Saskatchewan River, two parallel logs become tightly pressed together. They drift at the same speed as the water, but the current makes them rotate around their respective axes with a constant angular speed, as shown. At the top of the right log, a mouse of negligible mass moves with a constant speed of v=1cm/s relative to the log, such that the mouse remains stationary with respect to the water. At the top of the left log sits a muskrat of negligible mass, determined to catch and eat the mouse. The muskrat moves at a constant speed of $2v$ relative to the log, and sets off towards the mouse. If the circumference of each log is 60cm, how long does it take the muskrat to catch the mouse? (Answer in seconds.)

(a) 10.0
(b) 12.0
(c) 15.0
(d) 18.0
(e) 20.0



let ω be the angular speed.

$R\omega = 1\,cm/sec. = v$

both cylinders will have the same ω

distance $= \frac{2\pi R}{4} = d. = \frac{60}{4} = 15$

$v = \frac{d}{t}$ $\therefore$ $t = \frac{d}{v}$ in general.

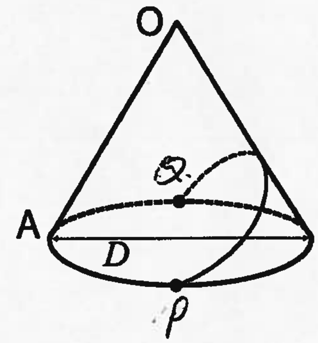
for left log : $t_1 = \frac{d}{2v - v} = \frac{d}{v} = \frac{15}{1} = 15$ sec.

for Right log $t_2 = \frac{d}{2v+v} = \frac{d}{3v} = \frac{15}{3(1)} = 5$ Sec

$\therefore$ total time $t = t_1 + t_2 = 20$ sec

answer (e)

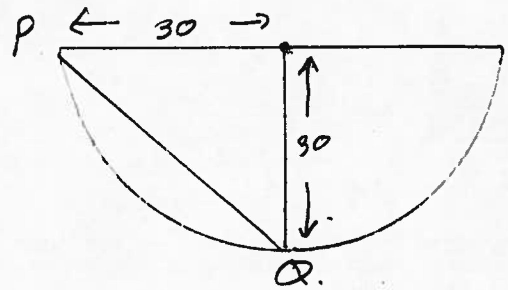
6) To help guide vehicles during the treacherous ice storm of 2013, a Mountie put down traffic cones with base diameter D = 30cm, and edge length OA = D. One cone became completely coated with frictionless ice of negligible thickness. The Mountie picked up the cone and stretched a massless elastic string between opposite sides of the base, as shown. If the string can slide freely along the frictionless surface, what was its equilibrium length? (Answer in cm.)
(a) 21.2 **(b)** 23.6 **(c)** 30.0 **(d)** 42.4 **(e)** 84.8



If you create a cone and cut along say OA you will obtain a flat shape as shown.

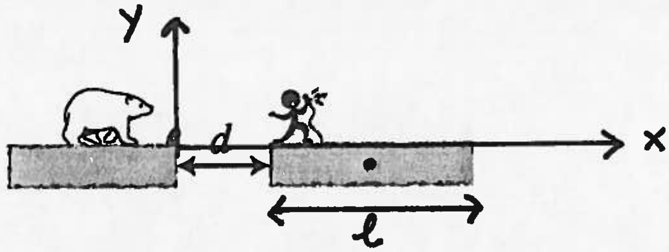
$\therefore PQ = \sqrt{30^2+30^2}$
$= 42.4$ cm.

answer (d)



Note:- The shortest distance is a straight line

7) Justin Bieber (mass 60kg) is singing a sold out show in Churchill, Manitoba. The town has constructed a stage out of ice floes on Hudson Bay, and he is standing at the edge of one (mass 180kg) while performing. Looking behind him, Justin sees a hungry polar bear stalking him from the neighboring floe, with gap distance of d=3m separating them. Realizing that the locals have set a trap for him, Justin tries to escape by running away from the bear at a constant speed of 4m/s relative to his floe. This causes his floe to drift and collide with the bear's floe (neglect water friction). At the instant of collision, the polar bear steps across to Justin's ice floe and starts to run from rest with a constant acceleration of $0.5 m/s^2$ (relative to Justin's flow). What is the *relative* speed between the bear and Justin when the polar bear finally pounces on him? (Answer in m/s.)



(a) 1.2
(b) 2.1
(c) 3.2
(d) 4.7
(e) 5.3

We assume the floe is uniform, the Bear and J.B are at the edges of each floe.

There are no net external forces acting hence the centre of mass is conserved (measured from origin).

$(X_{cm})_i = (X_{cm})_f$ M = mass of floe, m = mass of J.B and M' = mass of Bear.

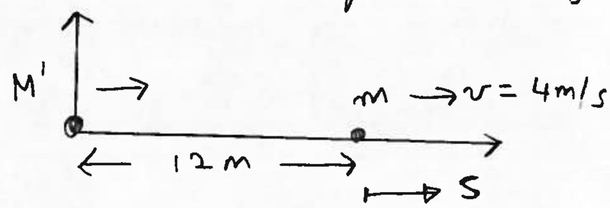
$$(X_{cm})_i = \frac{\sum m_i \bar{x}_i}{\sum m_i} \Rightarrow \frac{m(3) + M(\frac{\ell}{2}+3) + M'(0)}{m + M + M'}$$

$$(x_{cm})_f = \frac{m(x) + M(\ell/2) + M'(0)}{m + M + M'}$$

$(X_{cm})_i = (X_{cm})_f \Rightarrow x = 12\ m.$

∴ when the floes are touching J.B is 12 from the origi.

now from this instant say Bear gets J.B in some "t" seconds.



for J.B using $\bar{s} = \bar{u}t + \frac{1}{2}\bar{a}t^2 \rightarrow$

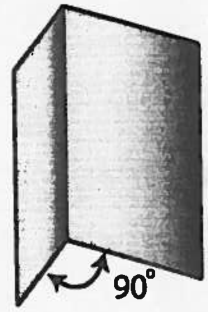
$S = 4t$ —① for Bear $(S+12) = 0(t) + \frac{1}{2}(0.5)t^2$ —②

②-① $\Rightarrow 12 = \frac{1}{4}t^2 - 4t \Rightarrow t^2 - 16t - 48 = 0 \Rightarrow t = 18.58$ sec

∴ $V_{Bear} = V_0 + at = 9.29$ m/s ∴ relative speed $= 9.29 - 4 = 5.29$

∴ answer is (e)

8) In fear, Batman is trying to escape from Robin, who just laid an egg. He climbs up the corner formed by the intersection of two vertical perpendicular walls. This feat requires a coefficient of static friction greater than one. Assuming $\mu_s = 1.5$, find the minimum magnitude of force with which Batman has to push on each wall while climbing. (Assume the magnitude of the force on the two walls is equal.) Express your answer in terms of Batman's weight, W.



(a) $0.41W$ (b) $0.81W$ (c) $1.03W$ (d) $1.62W$ (e) $2.01W$

f = "y" component of the frictional force

f' = "x" component of the frictional force

N = Normal force.

Since the acceleration is zero the Net force is also zero

$\Sigma \bar{F}_y = 0 \uparrow \Rightarrow 2f - Mg = 0 \Rightarrow f = Mg/2$

$\Sigma F_{horizontal} = 0 \Rightarrow f' - N = 0 \Rightarrow f' = N$

maximum static friction $= \sqrt{f^2 + f'^2} = f_s$

$f_s < \mu_s N \Rightarrow \sqrt{(\frac{1}{2}Mg)^2 + N^2} < \mu_s N \Rightarrow N > \frac{\frac{1}{2}Mg}{\sqrt{\mu_s^2 - 1}}$

$\therefore \mu_s > 1$

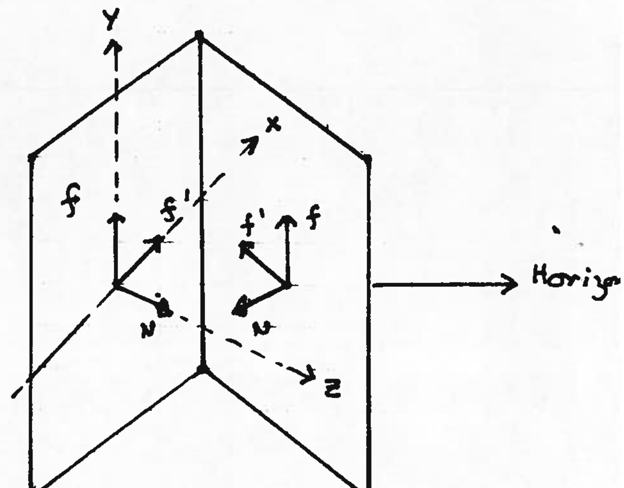
The Net force on each wall

$F = \sqrt{f^2 + f'^2 + N^2}$

$= \sqrt{(\frac{1}{2}Mg)^2 + N^2 + N^2}$

$\therefore F \geq \sqrt{(\frac{1}{2}Mg)^2 + 2\left[\frac{\frac{1}{2}Mg}{\sqrt{\mu_s^2 - 1}}\right]^2}$

$F \geq \sqrt{\frac{\mu_s^2 + 1}{\mu_s^2 - 1}}\,(\frac{1}{2}Mg)$ since $\mu_s = 1.5$



$F \geq \frac{1.61}{2} Mg$

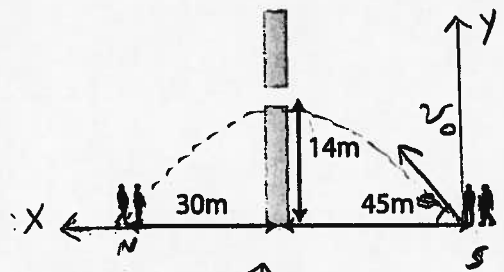
$F \geq 0.806\, Mg$

$\therefore F \geq 0.81 Mg$

answer (b)

9) While on a sovereignty patrol north of the 70th parallel, two bands of Conservative MPs are separated by a high ice wall. The company 30m north of the wall needs a pair of binoculars to track the movements of a suspicious herd of (possibly foreign) caribou. The South company is 45m south of the wall, and there is a hole just large enough to throw the binoculars through, 14m above the base of the wall. The South company has a mighty slingshot that can hurl the binoculars at great speed. Will the South company be able to get the binoculars to the North company, and if so, what speed must they be launched at? (Answer in m/s).

(a) It cannot be done
(b) 21.7
(c) 27.6
(d) 33.4
(e) 41.2



using $(y-y_0) = v_{0y} t + \frac{1}{2} a_y t^2$ ↑

$14 = v_0 \sin\theta\, t - 4.9 t^2$ — ① (assume $a_y = g = 9.8\ m/s^2$ ↓)

$(x-x_0) = v_{0x} t + \frac{1}{2} a_x t^2 \Rightarrow 45 = v_0 \cos\theta\, t$ — ②

in the same way for the full distance ↑ $0 = v_0 \sin\theta\, t' - 4.9 t'^2$

$75 = v_0 \cos\theta\, t'$

$\therefore t' = \frac{v_0 \sin\theta}{4.9} \Rightarrow 75 = v_0 \cos\theta \, \frac{v_0 \sin\theta}{4.9}$

$\therefore v_0^2 \sin\theta \cos\theta = (75)(4.9)$ — ③

② $\Rightarrow t = \frac{45}{v_0 \cos\theta}$ sub into ① $\Rightarrow 14 = v_0 \sin\theta \frac{45}{v_0 \cos\theta} - \frac{4.9\,(45)^2}{v_0^2 \cos^2\theta}$

$\therefore 14 = \frac{v_0^2 \sin\theta \cos\theta (45) - (4.9)(45)^2}{v_0^2 \cos^2\theta}$

$\Rightarrow 14 v_0^2 \cos^2\theta = 45 v_0^2 \sin\theta \cos\theta - (4.9)(45)^2$

$\therefore 45 v_0^2 \sin\theta \cos\theta = 14 v_0^2 \cos^2\theta + (4.9)(45)^2$ now using ③

$(45)(75)(4.9) = 14 v_0^2 \cos^2\theta + (4.9)(45)^2$

$\therefore v_0 \cos\theta = 21.73.$ — ④

sub into ② $\Rightarrow 45 = (21.73)\, t \Rightarrow t = 2.07$ sec.

sub into ① $\Rightarrow v_0 \sin\theta = 16.91.$ — ⑤

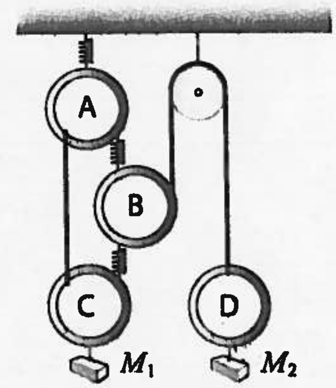
from ④ and ⑤ $\tan\theta = 0.78$ $\therefore \theta = 37.87^\circ$

and $v_0 = 27.53$ m/s ≈ 27.6 m/s

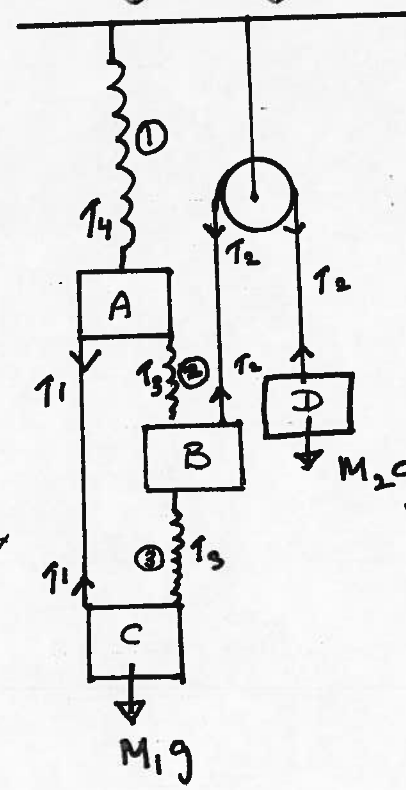
$\therefore$ answer is c

10) Vladimr Putin is constructing an elaborate spring and pulley system in order to hoist the Olympic rings at the opening ceremonies in Sochi. The system consists of rings A, B, C and D, which are connected by two unstretchable massless ropes (one of which passes over a frictionless pulley), and three massless springs with constant k = 50N/m. Intent on sabotaging the ceremony, Barack Obama stealthily attaches bricks to the bottom two rings (with M_1 = 0.6kg and M_2 = 0.5kg as shown in the diagram). Find the resulting displacement of ring D. (Answer in cm.)

(a) 1.1 **(b)** 1.5 **(c)** 1.9 **(d)** 2.9 **(e)** 4.9



Spring #1 extends Δy
Spring #2 compresses Δx.
$\therefore$ D would move $|\Delta y - \Delta x|$
A, B, C & D are massless pulleys.
Since additional masses M_1 & M_2 are attached.
let say $M_C = 0.6\,kg$ & $M_D = 0.5\,kg$



system is in equilibrium $\therefore \Sigma \vec{F} = 0$
for "D" $\Sigma \vec{F} = m\vec{a} = 0 \Rightarrow M_D g - T_2 = 0$ — ①
for "B" $\Rightarrow T_2 - 2T_3 = 0$ — ②
spring ② compresses & ③ extends by some amount
$\therefore$ for C $\Rightarrow T_1 + T_3 = M_C g$ — ③
for A $\Rightarrow T_4 - T_1 + T_3 = 0$ — ④
① $\Rightarrow T_2 = (0.5)g$ sub into ②
$\Rightarrow T_3 = \left(\frac{0.5}{2}\right) g$
$T_3 = k\Delta x$ — ⑤ $\therefore \Delta x = 0.049\,m \approx 5\,cm$

④ $\Rightarrow T_4 = M_C g - 2T_3 = 0.1g = k\Delta y$
$\therefore \Delta y = \frac{(0.1)(9.8)}{50} = 0.0196\,m \approx 2\,cm.$

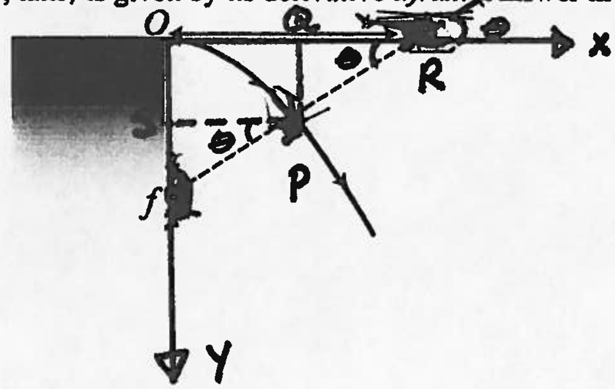
$\therefore |\Delta y - \Delta x| = 5 - 2 = 3\,cm \approx 2.9\,cm.$

answer is (d).

11) A skier accidentally veers off the edge of a sheer cliff (at point O in the diagram). His initial horizontal velocity is 30m/s, and he free-falls along a parabolic trajectory without air resistance. David Suzuki happens to be at the same cliff making a nature documentary. Upon seeing the skier fall, he jumps in his helicopter and flies horizontally from point O. Suzuki keeps a mountain goat (located on the cliff face at the focus of the parabola, f) in the background of his video, directly behind the falling skier. How far from point O is Suzuki after 2.5 seconds have elapsed?

Notes: (1) the equation for a parabola is $4fy=x^2$; (2) recall the trig identity $2\tan\theta = (1-\tan^2\theta)\tan(2\theta)$; (3) the slope of a curve, $\tan\theta$, is given by its derivative dy/dx. (Answer in m.)

(a) 148
(b) 164
(c) 188
(d) 204
(e) 226



given $x^2 = 4fy$ — (A)

$v_{0x} = 30\,m/s$

$(x - x_0) = v_{0x}t + \frac{1}{2}a_x t^2 \Rightarrow$

$x = 30t$ — (1)

Vertical motion ↓ $(y - y_0) = v_{0y}t + \frac{1}{2}a_y t^2$

$y = \frac{1}{2}gt^2$ — (2) $\quad$ (1) $\Rightarrow t = \frac{x}{30}$ $\therefore t^2 = \frac{x^2}{(30)^2}$

(2) $\Rightarrow t^2 = \frac{2y}{g} = \frac{x^2}{(30)^2}$ $\quad \therefore x^2 = \left[\frac{2(30)^2}{g}\right]y$ — (B)

Compare (A) & (B) $\Rightarrow f = \frac{2(30)^2}{4g} = \frac{v^2}{2g} = 45.9$

now $\tan\theta = \frac{Sf}{SP}$ but $OQ = SP = x = (30)(2.5) = 75$

from (2) $\Rightarrow OS = \frac{1}{2}(9.8)(2.5)^2 = 30.63$

$\therefore Sf = 45.9 - 30.63 = 15.28$

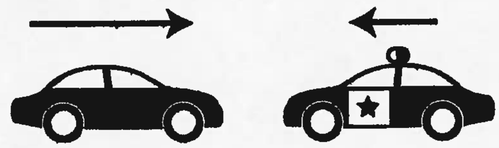
$\therefore \tan\theta = \frac{15.28}{75} = 0.2 = \frac{Of}{OR} \Rightarrow OR = \frac{Of}{0.2} = 225.4$

$\therefore OR \approx \underline{226\,m}$

answer (e)

12) **A police car moving at 100km/h sees Alexander Ovechkin in a Ferrari moving in the opposite direction with a relative speed that increases from 100km/h to 200km/h. When Ovechkin sees the cop, he accelerates, increasing the relative speed from 200km/h to 300km/h. What is the ratio of fuel used by the Ferrari in the second stage of acceleration with respect to that used in the first stage?**

(a) 3/1
(b) 5/3
(c) 5/1
(d) 7/3
(e) 7/5



The situation is equivalent to the observer at rest and the car starting from rest.

The ratio of the fuel used will be

$$\frac{K_2 - K_1}{K_1 - K_0}$$ where K_0 = The initial kinetic Energy
K_1 = The kinetic Energy after 1st Stage
K_2 = The kinetic Energy after 2nd Stage.

$K_0 = \frac{1}{2} m v_0^2$, $K_1 = \frac{1}{2} m v_1^2$, $K_2 = \frac{1}{2} m v_2^2$.

$$\therefore \frac{K_2 - K_1}{K_1 - K_0} = \frac{\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2}{\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2} = \frac{v_2^2 - v_1^2}{v_1^2 - v_0^2}$$

For the observer at rest $v_0 = 0$ km/hr
$v_1 = 100$ km/hr
$v_2 = 200$ km/hr.

$$\therefore \text{Ratio} = \frac{4v_1^2 - v_1^2}{v_1^2} = \underline{\underline{3}}.$$