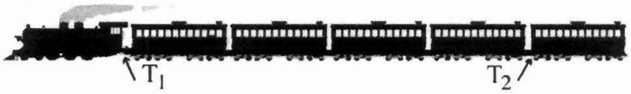
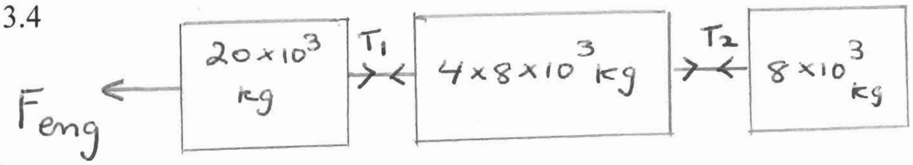


1) Some Canadian hockey fans get on a train in Toronto to go to the Olympics in Vancouver. The train consists of 5 carriages, each of mass 8,000 kg, when loaded, pulled by an engine of mass 20,000 kg. It accelerates uniformly from rest in Union Station, when the engine exerts a constant thrust of 3.0 kN. During this period, calculate $T_1 - T_2$, the difference in the tensions of the couplings T_1 (between the engine and the first carriage) and T_2 (between the final two carriages). Answer in kiloNewtons (kN).

(a) 0
(b) 1.0
(c) 1.6
(d) 2.2
(e) 3.4



$\sum \vec{F} = m\bar{a} \leftarrow$

$F_{eng} - T_1 = 20 \times 10^3 a$ — ①

$T_1 - T_2 = 4 \times 8 \times 10^3 a$ — ②

$T_2 = 8 \times 10^3 a$ — ③

also $F_{eng} = 3 \times 10^3$ N

$\therefore$ ① + ② + ③ $\Rightarrow F_{eng} = 60 \times 10^3 a$

$\therefore a = \frac{1}{20}$ m/s²

Sub into ② $\Rightarrow T_1 - T_2 = 4 \times 8 \times 10^3 \times \frac{1}{20}$

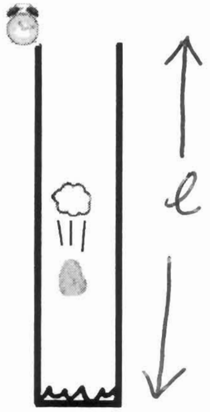
$= 1600$ N

$\therefore T_1 - T_2 = 1.6$ kN

2)Little Willie went to a well,
How deep it was, thought he could tell;
He threw a stone and timed the splash,
Then dropped his clock and heard it crash!

Willie dropped the stone from rest vertically from the top of the well and simultaneously started the stop clock (fortunately undamaged!). He stopped the clock the instant he heard the splash in some water at the bottom of the well, exactly 5.0 s later. Find the depth of the well in metres, given that the speed of sound in air is 330 m/s.

(a) 110
(b) 120
(c) 210
(d) 360
(e) 1650



Let t_1 be the time taken to create a splash.
object starts from rest

$v_{oy} = 0$ using

$(y - y_0) = v_{oy} t + \frac{1}{2} a_y t^2$ ↓

$y_0 = 0$, $y = \ell$, $a_y = 9.8\ m/s^2$ ↓

we are assuming down ↓ is the positive direction.

$\ell = 4.9 t_1^2$

time taken for the sound to travel back to the top $t_2 = \frac{\ell}{v}$ where v = speed of sound

$t_2 = \frac{\ell}{330}$ now $5 = t_1 + t_2$

$\therefore\ 5 = \sqrt{\frac{\ell}{4.9}} + \frac{\ell}{330}$

$\frac{\ell}{330} + \frac{\sqrt{\ell}}{2.21} - 5 = 0$ let $\sqrt{\ell} = x$.

Then we have a quadratic of the form.

$.003 x^2 + 0.452 x - 5 = 0$

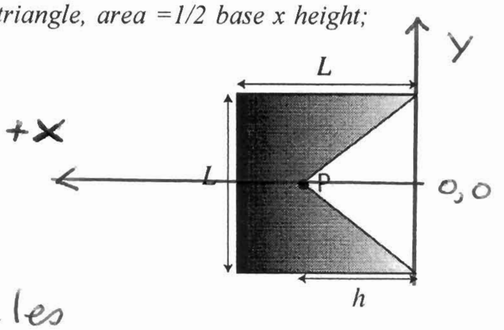
$x = \frac{-0.452 \pm \sqrt{(0.452)^2 - 4(.003)(-5)}}{2(.003)}$

$x = 10.35$

$\therefore\ \ell = 107.14\ m. \approx 110\ m.$

3)As part of Canada's economic stimulus plan, Stephen Harper is put to work sawing a piece of plywood to build a doghouse. He removes a triangle from the uniform, square sheet of side L, so that the center of mass (C.M.) of the remaining piece is at the apex P of the triangle. What is the height h of the triangle? [*For a triangle, area =1/2 base x height; C.M. is 2/3 height from the apex.*]

(a) 0.251 L
(b) 0.333 L
(c) 0.423 L
(d) 0.500 L
(e) 0.634 L



Center of mass (C.M) for n particles along the x axis is

$$(X_n)_{cm} = \frac{\sum_{i=1}^{n} m_i \bar{x}_i}{\sum_{i=1}^{n} m_i}$$

∴ for a 3 particle system

$$(\bar{x}_3)_{cm} = \frac{\sum_{i=1}^{3} m_i \bar{x}_i}{\sum_{i=1}^{3} m_i} = \frac{m_1\bar{x}_1 + m_2\bar{x}_2 + m_3\bar{x}_3}{m_1+m_2+m_3}$$

now say if we remove one particle (say m_2).

Then $$(x_2)_{cm} = \frac{(\bar{x}_3)_{cm} M_3 - m_2\bar{x}_2}{M_3 - m_2}$$

where $M_3 = m_1 + m_2 + m_3$

now we could think of having 2 uniform symmetrical objects, a square(S) and a Triangle(T). Due to symmetry the c.m will be on the x axis.

If M_S = mass of square & m_T = mass of triangle

$$X_{cm} = \frac{M_S (X_S)_{cm} - m_T (X_T)_{cm}}{M_S - m_T} = h$$

$$= \frac{\rho L^2 (L/2) - \frac{1}{2} L h \rho (1/3\, h)}{(L^2 - 1/2\, L h)\rho} = h$$

#(3) Continued.

Simplify the equation

$$\frac{L^2}{2} - \frac{1}{6}h^2 = Lh - \frac{h^2}{2}$$

$$3L^2 - 6hL + 2h^2 = 0$$

Compare this to $ax^2 + bx + c = 0$, where $x = h$

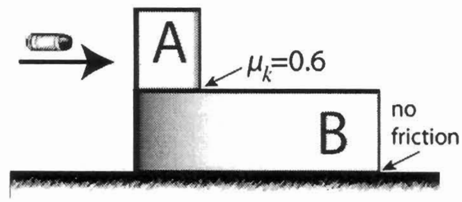
$$\therefore\ h = \frac{6L \pm \sqrt{36L^2 - 4(2)(3L^2)}}{2(2)}$$

$$= 2.36L \text{ or } 0.634L$$

$$\therefore\ h = 0.634L$$

4)Speed Skater Clara Hughes is creating a new event for the 2018 Olympics called the "Diathalon". Just like the skiers, the skaters sprint with a gun on their backs, and shoot at specially designed targets when arriving at a designated spot on the Oval. The target is an object (Block A) placed on a block of ice (Block B) which in turn sits on the Oval ice. The skater must shoot a horizontal bullet at Block A. A judge then measures the distance Block B travels before Block A stops sliding on Block B. As a trial, assume Block A has a mass of 4kg, Block B a mass of 1kg and the bullet a mass of 20 grams. The bullet is fired at 700 m/s into the top block, where it lodges. The coefficient of kinetic friction between the two blocks is 0.6. What will be the displacement of block B at the time block A comes to rest again on its surface? Assume the impulse from the bullet is enough to overcome static friction and start block A sliding. Also assume block B is long enough that A does not fall off the edge.

(a) 10.9 cm
(b) 16.5 cm
(c) 28.4 cm
(d) 31.1 cm
(e) 46.2 cm



The bullet collides with mass A.
This is a completely in elastic collision.
∴ momentum is conserved.

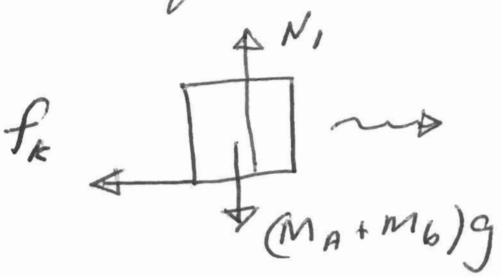
$$\therefore m_b \bar{v}_b = (m_A + m_b)\bar{v} \longrightarrow$$

$$\bar{v} = \frac{m_b v_b}{m_A + m_b} = \frac{[(20)10^{-3}][700]}{[4 + 20\times10^{-3}]}$$

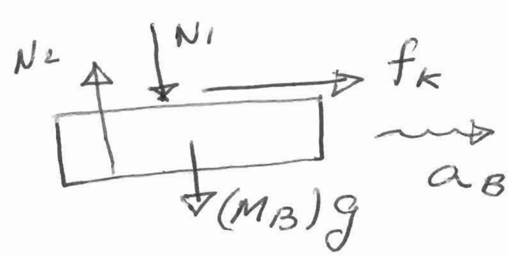
$$= 3.48 \text{ m/s}$$

now the mass A and bullet will slide on block B

f.b.d for (A + bullet)



f.b.d. for B



#4 Continued

The frictional force f_k will cause mass B (M_B) to slide. This will happen as long as A is sliding on B.

When A comes to rest on B (relative to B) B will have some speed $v_B \rightarrow$

Conservation of momentum.

$$\Sigma \vec{p_i} = \Sigma \vec{p_f} \longrightarrow$$

$$(m_A + m_b) v + m_B(0) = (m_A + m_B + m_b) v_B$$

$$\therefore v_B = \frac{(4.02)(3.48)}{5.02} = 2.79 \text{ m/s.}$$

$$f_k = \mu_k (m_A + m_b) g = (0.6)(4.02)(9.8)$$
$$= \underline{\underline{23.64 \text{ N}}}$$

$$\therefore \Sigma \vec{F} = m\vec{a} \text{ on } B \rightarrow, \quad f_k = (m_B) a_B$$

$$a_B = 23.64 \text{ m/s}^2.$$

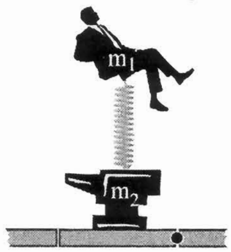
$$\therefore \text{using } v^2 - v_0^2 = 2a(x - x_0) \longrightarrow \text{for } B$$

$$(2.79)^2 - 0 = 2(23.64)(x - 0).$$

$$\therefore x = 0.165 \text{ m.}$$
$$= \underline{\underline{16.5 \text{ cm.}}}$$

5)Barack Obama is disgruntled with his campaign manager over recent polls in the US. Not sure what to do, Obama comes across a special punishment device in the basement of the White House, that Dick Cheney forgot to take with him. It consists of placing the subject on a massless vertical spring that is resting on an anvil with mass m_2=100 kg. The whole apparatus, (manager m_1=50 kg, plus spring plus block) rests on a trap door as shown. The trap door is then suddenly jerked down. Find the accelerations of the manager and the block immediately after the trap door is removed, in terms of **g**.

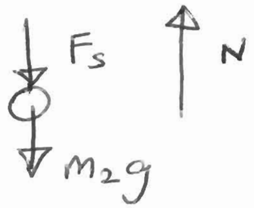
(a) a_1=0, a_2=1.5 g
(b) a_1=g, a_2=g
(c) a_1=0, a_2=g
(d) a_1=g, a_2=0
(e) a_1=0.33g, a_2=0.67 g



f.b.d. just before the door opens.

m_1

for m_2



$\Sigma F_y = m a_y$ ↓

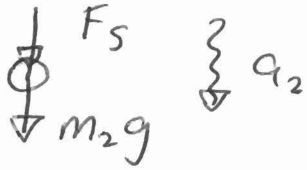
$m_1 g - F_s = 0$

$\therefore F_s = m_1 g$

now just an instant after the door is opened.

f.b.d for m_1 does not change

for m_2



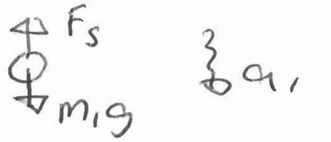
$\therefore F_s + m_2 g = m_2 a_2$

$(m_1 + m_2) g = m_2 a_2$

$\therefore a_2 = \frac{(m_1 + m_2) g}{m_2}$

$a_2 = 1.5 g$

for m_1

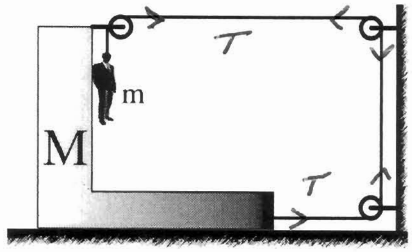


$F_s - m_1 g = m_1 a_1 = 0$

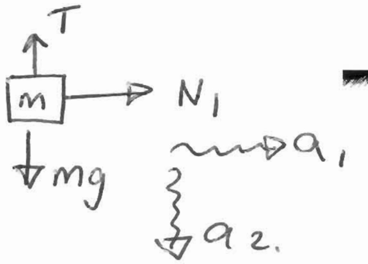
$\therefore a_1 = 0$

6)Gordon Campbell (BC's Premier) wants to improve his image after the uproar over destroying habitat for the Olympics. He proposes the display shown in the figure to be a part of the closing ceremony. He is tied onto the rope, and slides down the side of an icy block as the block moves - considered an "appropriate" act of atonement. This, of course is accompanied by an impressive display of fireworks, and a performance by a famous rapper. The block of mass M is free to move on the icy surface beneath it, and there is no friction anywhere. The rope and pulleys are massless. If Campbell's mass is $m=M/3$, find the magnitude of his total acceleration as he moves.

(a) 0.33 g
(b) 0.56 g
(c) 0.74 g
(d) 1.41 g
(e) 1.73 g



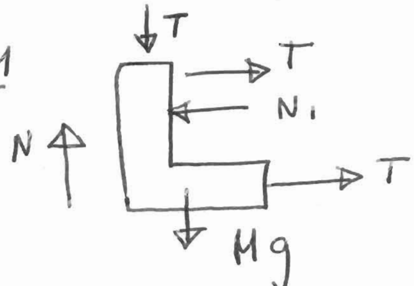
f.b.d for m



due to Conservation of string $\boxed{a_2 = 2a_1}$

$\Sigma \bar{F} = m\bar{a} \rightarrow \quad N_1 = m_1 a_1$ — ①

f.b.d for M



$\Sigma \bar{F} = m\bar{a} \rightarrow$

$-N_1 + 2T = (Ma_1)$ — ②

① + ② $\Rightarrow 2T = (M+m)a_1$ — ③

$\Sigma F_y = ma_y$ for m ↓

$mg - T = ma_2 = 2ma_1 \Rightarrow \quad 2mg - 2T = 4ma_1$ — ④

③ + ④ $\Rightarrow \quad 2mg = a_1(M + 5m)$

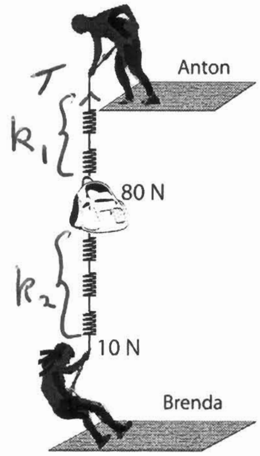
$\therefore \quad a_1 = \frac{(2m)g}{(M+5m)}$ If $M = 3m$

$a_1 = \frac{1}{4}g, \quad a_2 = \frac{1}{2}g \qquad a = \sqrt{a_1^2 + a_2^2}$

$= 0.56g$

7)Two mighty mountain climbers, Anton and Brenda, standing on two ledges one above the other, are pulling at two pieces of massless rope, attached to a pack that weighs 80 N. At Anton's side, the rope is attached to two massless springs in series, while at Brenda's side it is attached to three massless springs in series. All of the springs are identical. Brenda pulls down on her rope with a force of 10N. Find the force Anton must apply up on his rope to keep the pack in equilibrium.

(a) 30 N
(b) 80 N
(c) 90 N
(d) 110 N
(e) 290 N



Since springs are in Series.

$$\frac{1}{k_1} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k}$$

$\therefore k_1 = k/2.$

and $k_2 = k/3$

$\therefore$ The system looks as follows.

fbd for m_1

f.b.d for m_2.

$\Sigma F = ma \downarrow$

$$-T + k_1 x_1 = 0 \quad — ①$$

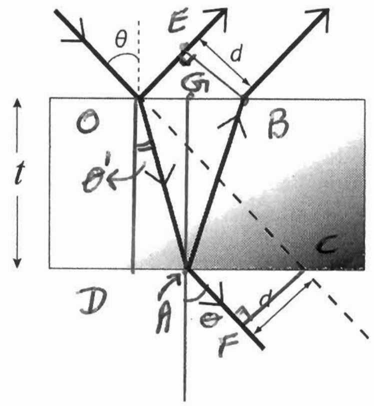
$$80N + k_2 x_2 - k_1 x_1 = 0 \quad — ②$$

$$10N - k_2 x_2 = 0 \quad — ③$$

$$① + ② + ③ \Rightarrow -T + 90N = 0 \Rightarrow T = 90N.$$

8)Cheech and Chong decide to shine a laser pointer onto a glass table top of thickness t in their smoky room. The smoke helps them observe two rays emerging from the top surface, one due to reflection, and the other due to refraction and internal reflection. In their state of mind they also realize that the two rays emerging from the top surface have the same separation d as the refracted ray at the bottom surface and the extension of the incident ray, as shown in the diagram. They know that glass has an index of refraction of 1.5 and the room they are in has an index of refraction of 1.0. Cheech challenges Chong to calculate the value of the incident angle θ with the information they have. If Chong is correct, what will the answer be in degrees?

(a) 33
(b) 44
(c) 56
(d) 61
(e) 67



$EB = CF$ is given.

If $D\hat{O}A = \theta'$ & $D\hat{O}C = \theta$.

$OB = 2t\tan\theta' = 2(OG)$

$OG = AD$

$AC = CD - AD$
$= t\tan\theta - t\tan\theta'$

Since $CF = EB$ $\therefore 2(OG) = AC$.

$OB \cos\theta = AC \cos\theta$

$\therefore 2t\tan\theta' = t(\tan\theta - \tan\theta')$

$\tan\theta = 3\tan\theta'$ — ①

using Snell's Law $\sin\theta = 1.5\sin\theta'$ — ②

① $\Rightarrow \dfrac{\sin\theta}{\cos\theta} = 3\dfrac{\sin\theta'}{\cos\theta'} \Rightarrow \dfrac{\sin\theta}{\sin\theta'} = 3\dfrac{\cos\theta}{\cos\theta'} = 1.5$

(using ②). $\therefore \cos\theta = (0.5)\cos\theta'$ — ③

Since $\sin^2\theta + \cos^2\theta = 1$, $②^2 + ③^2 \Rightarrow 1 = (1.5)^2\sin^2\theta' + (0.5)^2\cos^2\theta'$

$\therefore 1 = (1.5)^2(1-\cos^2\theta') + (0.5)^2\cos^2\theta'$

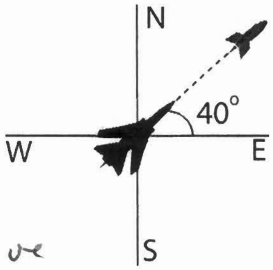
$1 = 2.25 - 2.25\cos^2\theta' + 0.25\cos^2\theta'$

$\therefore 2\cos^2\theta' = 1.25 \Rightarrow \theta' = 37.76^\circ$

and $\theta = 66.71^\circ \simeq 67^\circ$

9)A dilapidated CF-18 fighter jet has a cruising speed of 100 m/s relative to the air. Flying over the flat Saskatchewan prairies and experiencing a wind of speed 50 m/s from the North, a pilot is pointing the airplane at 40 degrees North of East at a constant height of 500m. Suddenly, the pilot fires a bullet horizontally from a gun pointing straight out of the nose of the plane, with a speed 200 m/s relative to the plane. Ignoring all air resistance on the bullet, what is the distance between the airplane and the bullet when it finally hits the wheat fields below?

(a) 1950 m
(b) 2005 m
(c) 2020 m
(d) 2081 m
(e) 2733 m



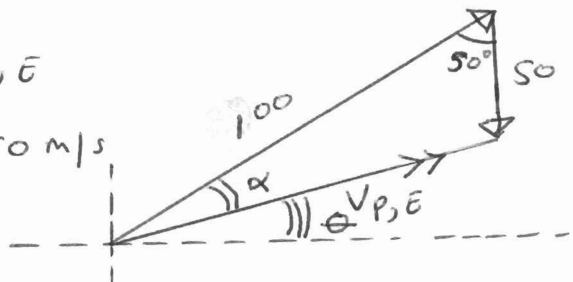
$\bar{V}_{p,w}$ = Velocity of plane relative to wind (air = wind).

$\bar{V}_{w,E}$ = Velocity of wind relative to Earth.

$\bar{V}_{p,E}$ = Velocity of plane relative to Earth.

$\vec{V}_{p,E} = \vec{V}_{p,w} + \vec{V}_{w,E}$

200 m/s at 40°   ↓ 50 m/s



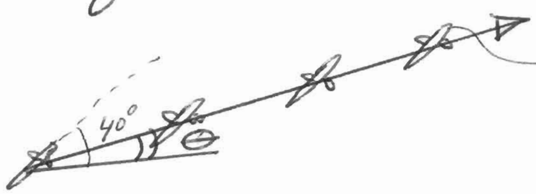
[Top view XY plane]

$V_{p,E} = \sqrt{100^2 + 50^2 - 2(100)(50)\cos 50}$ (Cosine Rule).

$= 77.9$ m/s.

using the Sine Law $\frac{50}{\sin\alpha} = \frac{77.9}{\sin 50} \Rightarrow \alpha = 29.4^\circ$, $\theta = 10.6^\circ$

We say we have a motion that could be described by the following drawing.



now a bullet is shot relative to the plane.

$\bar{V}_{B,E}$ = Velocity of the bullet relative to the Earth.

$V_{B,P}$ = Velocity of the bullet relative to the plane

200 at 40°

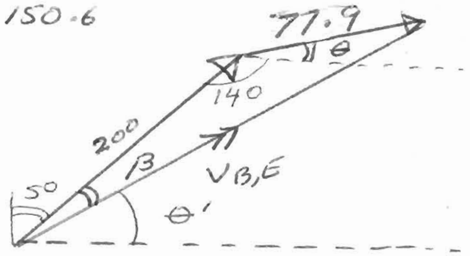
#9 Continued

$$\vec{V}_{B,E} = \vec{V}_{B,P} + \vec{V}_{P,E}$$

[Top View XY plane]

$$= \sqrt{200^2 + 100^2 - 2(200)(77.9)\cos 150.6}$$

$$= 270.58 \text{ m/s}$$



using Sine Law

$$\frac{77.9}{\sin\beta} = \frac{270.58}{\sin(150.6)} \Rightarrow \beta = 8.1^\circ$$

and $\theta' = 31.9^\circ$

now the bullet does not have any vertical (z direction) velocity component.

∴ the time taken for it to hit the ground

$(y - y_0) = v_{oy} t + \frac{1}{2} a_y t^2$ ↓ $a_y = 9.8 \text{ m/s}^2$ $y = 500 \text{ m}$.

∴ $500 = \frac{1}{2}(9.8) t^2 \Rightarrow t = 10.1 \text{ sec}$.

during this time the distance moved in the xy plane

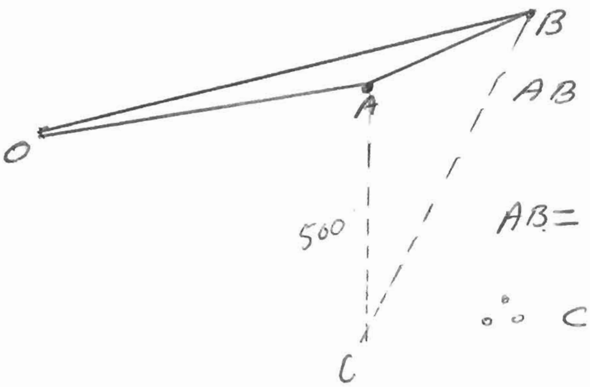
$(x - x_0) = v_{ox} t + \frac{1}{2} a_x t^2$ $a_x = 0$

$x = (270.58)(10.1) = 2733.27 \text{ m}$.

during this time the plane would have moved

$77.9 \times 10.1 = 786.79 \text{ m}$.

∴ the horizontal displacement between the plane and bullet in 10.1 sec would be.



$B\hat{O}A = 31.9 - 10.6 = 21.3^\circ$

$AB = \sqrt{2733.21^2 + 786.79^2 - 2(2733.21)(786.79)\cos 21.3}$

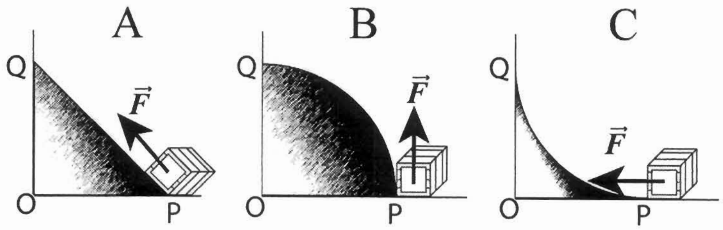
$AB = 2020.48$.

$\therefore CB = \sqrt{500^2 + 2020.48^2}$

$= 2081.43$

10)Jack Layton is desperately trying to return a crate of prorogued parliamentary bills that he found dumped in the Ottawa river. The crate can be pushed up the frictionless icy bank towards parliament hill along three different paths, from point P to Q, by a constant magnitude force *F* that is maintained parallel to the path's surface. P and Q are each located a distance R from the origin O. B and C have 90 degree circular arcs of radius R, but A has a straight line. If the magnitude of *F* is the same in all three cases, rank the work done in each case.

(a) $W_C > W_B > W_A$
(b) $W_B > W_C > W_A$
(c) $W_A = W_B = W_C$
(d) $W_B = W_C > W_A$
(e) $W_A > W_B = W_C$



Total work = work done by gravity plus work done by force.

for A, B, and C work done by gravity will be the same.

However work done by the force for A $= \Sigma \vec{F} \cdot d\vec{r} = F\sqrt{2}R = W_A$

Since $\vec{F}$ is parallel to the path.

work done by force for B & C $= F\frac{(2\pi R)}{4}$. once again since F is parallel to the path.

$\therefore W_B = W_C = \frac{F(2\pi R)}{4}$

But $W_B > W_A$ $\quad \therefore W_B = W_C > W_A$

11)Two balls are dropped from a large height h metres above the ground. The ball on top is small, with mass m_1, and the ball on the bottom heavy with mass m_2. Assume the lower ball collides elastically with the ground. Then, as the lower ball starts to move upward, it collides elastically with the upper ball, still moving downwards. How high will the upper ball rebound in the air? Assume that $m_2 >> m_1$.

(a) 0.5 h
(b) 3.5 h
(c) 5.0 h
(d) 7.5 h
(e) 9.0 h

The balls are dropped from a height "h" from rest. So when it just hits the ground the velocity v can be found by

$$v^2 - v_0^2 = 2g(h - h_0) \downarrow$$

$$v = \sqrt{2gh}$$

after m_2 bounces back m_2 is going up with speed v and m_1 comes down with speed v. since the collision is elastic momentum and energy are conserved.

$\Sigma \bar{p}_i = \Sigma \bar{p}_f \uparrow \quad m_2 v - m_1 v = m_1 v_1' + m_2 v_2'$ — ①

$(KE)_i = (KE)_f \quad \frac{1}{2} m_2 v^2 + \frac{1}{2} m_1 v^2 = \frac{1}{2} m_2 v_2'^2 + \frac{1}{2} m_1 v_1'^2$ — ②

∴ ① ⇒ $m_2 (v - v_2') = m_1 (v_1' + v)$ — ③

now ② ⇒ $m_2 (v^2 - v_2'^2) = m_1 (v_1'^2 - v^2)$

$m_2 (v - v_2')(v + v_2') = m_1 (v_1' + v)(v_1' - v)$

using ③ $m_1 (v_1' + v)(v + v_2') = m_1 (v_1' + v_2')(v_1' - v)$

$v + v_2' = v_1' - v$

$\boxed{v_2' = v_1' - 2v}$ — ④

11 continued.

now sub this into ①

$$m_2 v - m_1 v = m_1 v_1' + m_2 (v_1' - 2v)$$

$$m_2 v - m_1 v = m_1 v_1' + m_2 v_1' - 2 m_2 v$$

$$3 m_2 v - m_1 v = v_1' (m_1 + m_2)$$

$$\therefore \quad v_1' = \left(\frac{3 m_2 - m_1}{m_1 + m_2} \right) v$$

Since $m_2 \ggg m_1$ $\quad v_1' = \frac{3 m_2}{m_2} v = 3v$.

hence $\quad v_2' = v$

since we assumed up to be positive m_1 will bounce off m_2.

To find the maximum height it would travel we can use

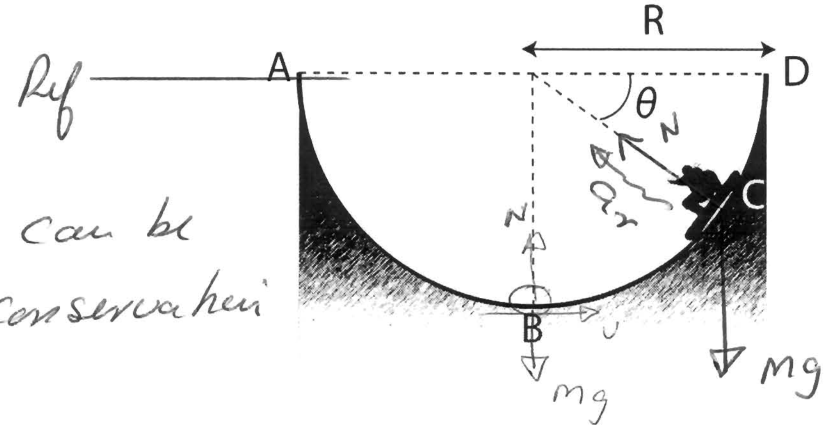
$$v^2 - v_0^2 = 2a(y - y_0). \quad \uparrow$$

$$0 - \left[3\sqrt{2gh} \right]^2 = 2(-g)y$$

$$\therefore 9h = y$$

12)Olympic Champion Shaun White practices at his personal half-pipe, a large semi-cylindrical surface kept frictionless with a smooth layer of ice. Shaun remains crouched on his snowboard, and releases from rest at point A. The resulting ride passes downward from A, follows the surface horizontally through B and coasts upwards through C to stop at D. (Then, it could oscillate forever, or until the ice melts, if we ignore air resistance). Using his mind and not his muscles, Shaun correctly calculates that at point "C" the magnitude of the total force exerted on him by the ice is exactly 40% of what it is at point "B". Calculate the value of the angle θ. Answer in degrees.

(a) 21.8
(b) 23.6
(c) 28.2
(d) 36.0
(e) 66.4



Speed at B can be found using conservation of energy.

$$\frac{1}{2} m v_B^2 = mgR$$
$$v_B^2 = 2Rg$$

$\Sigma F = ma$ at B

$$N - mg = \frac{m v_B^2}{R} \Rightarrow N = \frac{m}{R} 2Rg + mg = 3mg$$

Speed at C using conservation of energy

$$\frac{1}{2} m v_C^2 = mgR\sin\theta$$
$$v_C^2 = 2Rg\sin\theta$$

$\Sigma F = ma$ at C

$$N - mg\sin\theta = \frac{m v_C^2}{R}$$
$$N = mg\sin\theta + \frac{m}{R}(2Rg\sin\theta)$$

but $N = 0.4(3mg)$ $\therefore$ $(.4)(3mg) = mg\sin\theta + 2mg\sin\theta$

$$\therefore \sin\theta = 0.4$$
$$\theta = 23.6^\circ$$