SPH4U: Special Relativity

Course Website: http://mrohrling.yolasite.com

This syllabus contains a list of all classes, topics and homework in the Gr. 12 Kinematics Unit. You are strongly encouraged to explore the simulations and videos listed for each lesson – they are optional but quite interesting!

Day	Topics	Homework	Extras
1	Velocity and Frames of Reference		
2	The Light Clock	Handbook problems: #1-4, identify the types of intervals only, don't solve!	Active Physics: Time Dilation The Light Clock Video: Time Dilation Video: Special Relativity
3	The Moving Ruler	Handbook problems: #5, 6, identify the types of distances only, don't solve!	Active Physics: Length Contraction Video: Relativity Made Easy Video: Length Contraction Video: Visualizing Relativity
4	Al's Relativistic Adventure	Activity: Al's Relativistic Adventure (BYO headphones)	Video: Crash Course…Special Relativity You may have to add the "Flash Player emulator 2022' as an extention to your Chrome browser.
5	Relativity Problem Solving	Handbook problems: #1-7	
6	Energy and Relativity	Handbook problems: #8-10	Video: Mass and Energy Video: Einstein Talks
7	Energy and Relativity	Handbook problems: #11,12	Text: pg. 690-691 Video: Large Hadron Collider (LHC) Video: Proton Antiproton Collision
8	Relativity Problem Solving		
9	Relativity Test		

SPH4U: Velocity and Frames of Reference

Recorder: _____
Manager: _____
Speaker: _____
0 1 2 3 4 5

All physics quantities that we measure depend on the frame of reference of the observer. Alice is standing on the Earth and watches a train go by with a velocity of 150 km/h [E]. Inside the train stands Bob. Both Alice and Bob are physicists and make observations about each other's motion.

1. Complete the chart showing the measured velocity of each object from each reference frame.

Object	Frame A (Alice)	Frame B (Bob)
Alice		
Bob		
Earth		
Train		

2. Bob has a ball and throws it. He measures the velocity of the ball to be 40 km/h [E]. The train keeps going at its usual speed. Complete the chart showing the measured velocity of the ball from each reference frame. Explain how you found the velocity of the ball relative to frame A.

Object	Frame A (Alice)	Frame B (Bob)
Ball		

3. Bob throws a second ball and measures the velocity to be 30 km/h [W]. Complete the chart showing the measured velocity of the ball from each reference frame. Explain how you found the velocity of the ball relative to frame A.

Object	Frame A (Alice)	Frame B (Bob)
Ball		

4. Bob pulls out a flashlight, points it east and turns it on. Using a fancy apparatus he measures the velocity of a particle of light from his flashlight to be 300 000 000 m/s [E]. Using the previous logic, what is the velocity of the light relative to Frame A in m/s?

5. Imagine Bob was on an "express" train that travelled at 2×10^8 m/s [E] and turned on his flashlight just as in question 4. What is the velocity of the light relative to Frame A?

6. Alice now has her flashlight turned on and points it east. Bob's same express train passes by. What is the velocity of the light from Alice's flashlight relative to Bob?

SPH4U: Relativity Problems

1. A cosmic ray travels 60 km through the earth's atmosphere in 400 µs (10^{-6} s), as measured by experimenters on the ground. How long does the journey take according to the cosmic ray?

2. At what speed, as a fraction of *c*, does a moving clock tick at half the rate of an identical clock at rest.

3. An astronaut travels to a star system 4.5 ly away at a speed of 0.9*c*. Assume the times needed to speed up and slow down are negligible.
   (a) How long does the journey take according to Mission Control on Earth?
   (b) How long does the journey take according to the astronaut?

4. How fast must an astronaut travel on a journey to a distant star so that the astronaut ages 10 years while the Mission Control workers age 120 years?

5. Jill claims that her new rocket is 100 m long. As she flies past your house, you measure the rocket's length and find that it is only 80 m/ Should Jill be cited for exceeding the 0.5*c* speed limit?

6. A muon travels 60 km through the atmosphere at a speed of 0.9997*c*. According to the muon, how thick is the atmosphere?

7. Our Milky Way galaxy is 100,000 ly in diameter. A spaceship crossing the galaxy measures the galaxy's diameter to be a mere 100ly.
   (a) What is the speed of the spaceship relative to the galaxy?
   (b) How long is the crossing time as measured in the galaxy's frame of reference?

8. What are the kinetic energy, the rest energy and the total energy of a 1.0 g particle with a speed of 0.8*c*?

9. At what speed is a particle's kinetic energy twice its rest energy?

10. In an attempt to reduce the extraordinarily long travel times for voyaging to distant stars, some people have suggested travelling close to the speed of light. Suppose you wish to visit the red giant start Betelgeuse, which is 430 ly away, and that you want your 20 000 kg rocket to move so fast that you age only 20 years during the journey.
    (a) How fast must the rocket travel relative to the earth? (Hint: roughly how long will the journey take according to an observer on the earth?)
    (b) How much energy is needed to accelerate the rocket to this speed?
    (c) Compare this amount of energy to the total used by the United States in the year 2010, which was roughly $1.0\text{x}10^{20}$J.

11. The nuclear reaction that powers the sun is the fusion of four protons ($1.673\text{x}10^{-27}$ kg) into a helium nucleus ($6.645\text{x}10^{-27}$ kg). The process involves several steps, but the net reaction is simply: 4p $\rightarrow$ He + energy. How much energy is released overall in each fusion process? Give your answer in J and MeV.

12. Consider the completely inelastic collision $e^- + e^- \rightarrow e^- + e^+ + e^- + e^+$ in which an electron-positron pair is produced at rest in a head-on collision between two electrons moving in opposite directions at the same speed.
    (a) What is the minimum kinetic energy each electron must have to allow this process to occur?
    (b) What is the speed of an electron with this energy?

1. 346 µs
2. 0.866*c*
3. (a) 5 yr, (b) 2.18 yr
4. 0.996*c*
5. Yes
6. 1.47 km
7. (a) 0.9999995*c*, (b)100 000.05 yr
8. $6.0\text{x}10^{13}$ J, $9.0\text{x}10^{13}$ J, $1.50\text{x}10^{14}$ J
9. 0.943*c*
10. (a) 0.999*c*, (b) $3.69\text{x}10^{22}$J, (c) 370 times greater!
11. $4.22\text{x}10^{-12}$J, 26.4 MeV
12. (a) $8.19\text{x}10^{-14}$ J, (b) 0.866*c*

From R. Knight, *Physics for Scientists and Engineers. A Strategic Approach.*Pearson Education, 2013

SPH4U: The Light Clock

Recorder: _____
Manager: _____
Speaker: _____
0 1 2 3 4 5

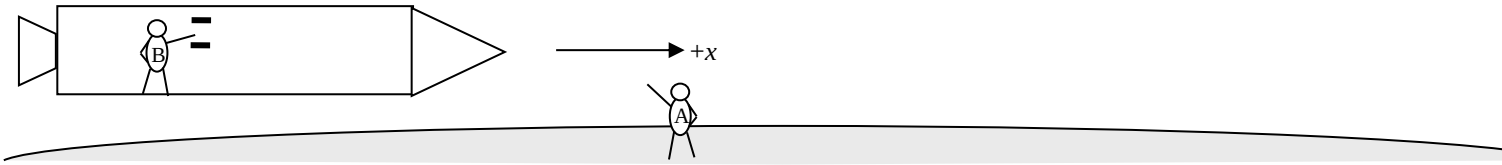
Einstein thought deeply about the train scenario we studied last class and suggested two ideas that change our understanding of what happens when Bob turns on his flashlight. Einstein postulated that:

(1) All observers, whether they are moving fast or slow in an inertial frame will observe light to travel at $c = 2.998 \times 10^8$ m/s in vacuum, and

(2) the laws of physics are the same for all inertial frames – there are no special rules if you are moving fast or slow.

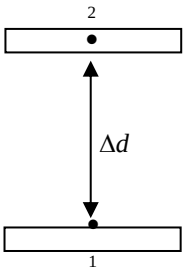
These suggestions are known as the two *postulates of special relativity*.

A: The World According to Bob



Bob is now travelling in a spacecraft at a velocity v relative to the earth. He is carrying with him a light clock – a special kind of clock imagined by Einstein. The clock consists of two perfect, smooth mirrors that face each other and are separated by a small distance Δd. A particle of light (a photon) reflects back and forth between the mirrors which are lined up carefully so that the photon always reflects off the same two points on the mirror's surface. We note two events that take place: event 1where the photon leaves the bottom mirror and event 2 where the photon reaches the top mirror. The time it takes for the photon to travel between the mirrors represents the 'tick' of the clock.

1. **Reason.** Use the symbols from the description of this situation to help complete the chart of measurements from Bob's frame of reference.



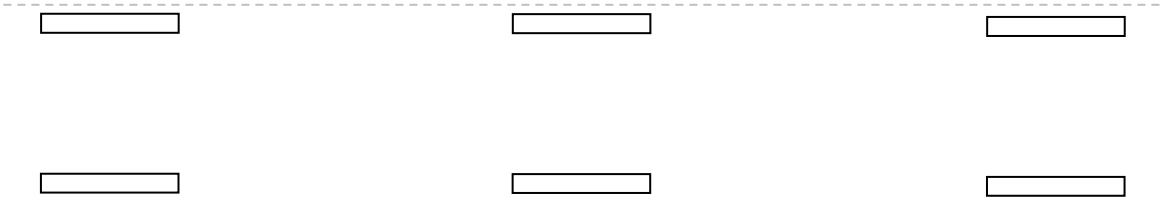
Time interval between events1and2 (one tick)	Δt_o
Velocity of the light clock	
Distance between events1and2	
Speed of the photon	

2. **Represent.** Construct an equation that relates the speed of the photon to the distance and time it travels as measured in Bob's frame.

B: The World According to Alice

Alice is standing on Earth watching Bob and his light clock travel by in the rocket ship. She is able to make careful measurements of the light clock and its photon.

1. **Represent.** From Alice's frame, we see the light clock at three moments in time corresponding to three events: the photon at 1, the photon at 2 and then the photon returning to the bottom mirror. You may assume the rocket is travelling quite fast! Draw the path of the photon through space. Label the distance between events 1 and 2 as ΔD.



2. **Reason.** Complete the chart of measurements from Alice's frame of reference. **No calculations are required!**

Time interval between events 1 and 2 (one tick)	Δt
Velocity of the light clock	
Distance between events 1 and 2	
Speed of the photon	

3. **Represent.** Construct an equation that relates the speed of the photon to the distance and time it travels as measured in Alice's frame.

4. **Reason.** Compare the size of the results from the two frames of reference.

Measurement	Comparison
Speed of photon	
Distance between 1 and 2 (Δd vs. ΔD)	
Time interval between 1 and 2 (Δt vs. Δt_o)	

Note that both observers **must** agree on the speed of light according to the first postulate of special relativity.

5. **Speculate.** What does your comparison imply about the flow of time on the spacecraft according to Alice?

C: Time Dilation

A direct consequence of Einstein's two postulates is that the time interval between two ticks of a clock is shortest when the clock is at rest relative to an observer. This is not an optical illusion, a delay effect, or a mechanical defect of the clock. The flow of time actually depends on who is observing and their speed. This idea is called *time dilation*. To help carefully describe time intervals we introduce two definitions:
Proper time (Δt_o): The time interval between two events that occur at the same position in space.
Relativistic time (Δt): The time interval between two events that occur at two different positions in space.
The relativistic time interval will always be greater than the proper time interval ($\Delta t > \Delta t_o$) as long as there is relative motion of the two observers. Each observer (and you too!) needs to decide **every time**, whether the time interval you are studying is proper or relativistic according to their own frame of reference. Gone are your days of innocence when time intervals were just time intervals!

1. **Explain.** Use the new definitions to help explain which type of time interval Alice and Bob measured. Note: we will assume that Δd is very small and can be ignored.

2. **Interpret.** Alice has a toaster which she starts. Bob measures a time interval of 72 s between the two events of starting the toaster and the 'pop' at the end. Alice measures a time interval of 60 s between the same two events on her light clock. Explain what type of time interval each observer measured.

The two different time intervals (as measured in different frames) between the same pair of events do not lead to any kind of logical contradiction. A contradiction in physics only occurs when two observers in **the same** frame of reference get different results. Measurements of the same events by different observers in a single reference frame should agree with one another. This is what we have been assuming all along in our study of physics!

To figure out how much larger the relativistic time interval is, we can do a mathematical analysis of the light clock and carefully compare ΔD with Δd. If we do this (try it – it's fun!) we get the time dilation equation: $\Delta t = \gamma \Delta t_o$, where $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$. We need to learn how to use the expression for γ (gamma), but that will come later.

SPH4U: The Moving Ruler

Recorder: _____
Manager: _____
Speaker: _____
0 1 2 3 4 5

Consider a subatomic particle called the muon (μ) which is moving rapidly at a speed v relative to a ruler in the direction of the ruler's length. We note two events: event 1 when the muon is at the 0 centimeter mark, and event 2 when the muon is at 30 centimeter mark. You need a ruler and an eraser (muon) for this investigation.

A: The Ruler's Frame

1. **Represent.** Act out this situation from the frame of reference of the ruler. Pretend to use a stopwatch. Say out loud, "1" and "2" when events 1 and 2 occur. **Demonstrate this for your teacher**. Sketch this situation and label the two events. Indicate the velocity of the muon v and the distance between the two events Δx_o.

2. **Reason.** An observer in the ruler's frame of reference measures the time interval between the two events. Carefully explain what type of time interval this is. Act this out by showing when you start and stop your pretend stopwatch. What symbol should you use to represent it?

3. **Represent.** Write an equation that relates speed, distance and time of the muon as measured by an observer in the ruler's frame.

B: The Muon's Frame

1. **Represent.** Act out this situation from the frame of reference of the muon. Say out loud, "1" and "2" when events 1 and 2 occur. **Demonstrate this for your teacher**. Sketch this situation and label the two events – clearly show that the ruler has moved! Indicate the velocity of the ruler v and the distance the ruler travels between events 1 and 2.

μ

2. **Reason.** An observer in the muon's frame of reference measures the time interval between the two events 1 and 2. Act this out by showing when you start and stop your pretend stopwatch. Carefully explain what type of time interval this is. What symbol should you use to represent it?

3. **Represent.** What distance does the metre stick travel in this frame of reference? Label this as Δx on the diagram.

4. **Represent.** Write an equation that relates speed, distance and time for the metre stick as measured by an observer in the muon's frame.

5. **Reason.** Compare the size of the results from each frame.

Speed of muon / metre stick	
Time between events 1 and 2	
Distance between events 1 and 2	

6. **Reason.** What can we conclude about the distance measurement of each observer?

C: Length Contraction

The second consequence of Einstein's two postulates is that the spatial interval between two events (the distance) also depends on the observer! Moving objects (or intervals of space) become smaller along their direction of motion. This is called *length contraction*. This is not an optical illusion – space itself (even if it's empty) contracts. So a ruler moving towards us contracts. If we travel past Earth, the space between Earth and the moon will contract. We define two different types of distances or lengths.

Proper length (Δx_o): The distance between two points (ends of an object, positions in space) that are at rest relative to an observer.

Relativistic length (Δx): The distance between two points (ends of an object, positions in space) that are moving relative to an observer.

The relativistic length is always smaller than the proper length ($\Delta x < \Delta x_o$). A quick calculation gives the expression:

$\Delta x = \Delta x_o/\gamma$

SPH4U: Relativity Problem Solving

Recorder: _____
Manager: _____
Speaker: _____
0 1 2 3 4 5

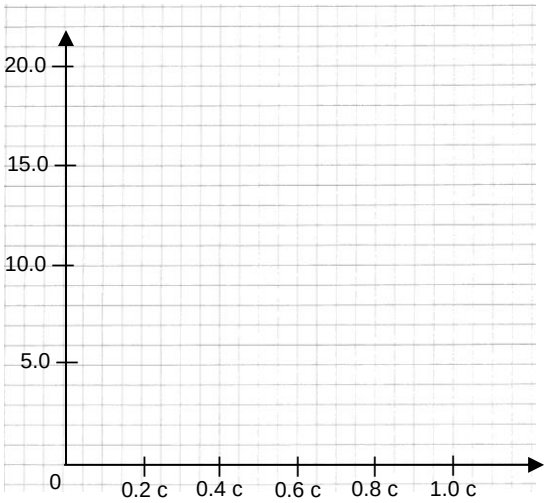
A: Why Don't We Notice?

The consequences of Einstein's two postulates seem really crazy to us largely because we have never noticed the changes to time and length intervals. We must now address this: why have we never noticed time slowing down or lengths contracting for drivers on the 401? The expression for gamma: $\gamma = (1 - v^2/c^2)^{-1/2}$ will help us to answer this question.

In special relativity, express your velocity values as a fraction of *c*. For example, $v = 1.5 \times 10^8$ m/s = 0.5 *c*. When you substitute the velocity written this way into γ, the *c*'s divide out nicely and the math is much friendlier.

1. **Calculate and Represent.** Complete the chart below. Rewrite the first five speeds in terms of *c*. Calculate γ for each speed. Sketch a graph of γ vs. *v*.

Speed (m/s)	**Speed (c)**	**γ**
Fast Runners, 10 m/s		
Fast Cars, 40 m/s		
Fast Jets, 600 m/s		
The Space Shuttle, 7 860 m/s		
Voyager Space Probe, 17 000 m/s		
	0.1 c	
TV screen electrons	0.3 c	
	0.5 c	
	0.7 c	
	0.9 c	
	0.99 c	
X-Ray Machine Electrons	0.999 c	
LHC protons, 0.999 999 999 95 c		



In relativity we often encounter extreme numbers. We need to judge significant digits by the digits which **are not zero** for γ (1.00007 has one useful digit), or which are not 9 for velocities in terms of *c* (0.9994*c* has one useful digit).

2. **Explain.** Should the first five γ values you calculate be *exactly* the same?

3. **Explain.** Based on the chart, offer a simple explanation for why relativistic effects are not noticed in daily life.

4. **Describe.** What happens to the size of γ as *v* approaches the value *c*?

5. **Reason.** What does this tell us about the flow of time for a highly relativistic object (speeds close to *c*)?

6. **Apply.** Relativistic effects are important for GPS satellites which orbit at a similar speed to the space shuttle relative to the ground. Precision timing is absolutely essential for determining an object's location on the earth. For a GPS satellite observed from the earth, $\gamma = 1.000\ 000\ 000\ 3$.
   a) One day (86400 s) ticks by on a clock in the GPS satellite. How much time does this take according to an observer on Earth? What is the difference in the two times? (It might help to use your phonulator turned sideways)

   b) How far does light travel in that time difference? The GPS system uses microwaves that travel at the speed of light to locate you. What would this mean for the reliability of the GPS system?

B: Relativity Problem Solving

Now that we have mathematical tools at our disposal, we are ready to solve the problems we have started over the past few lessons. Here are some important tips for the math part of our relativity work.

D: Mathematical Representation

- When possible, find a numerical value for γ first: this is a valuable check for your work. In multi-part problems, you might use γ more than once, so this simplifies your work
- A convenient unit of distance is the light year: 1 ly = c·a (speed of light × year). If your calculations involve light-years (ly) of distance and years of time (a = year), you do not need to convert anything, just replace the unit ly with c·a to show how the units multiply or divide out
- Remember to keep at least 4 significant digits during your math work and use 3 for the final answers

E: Evaluate

- Based on your understanding of relativistic or proper intervals, you should be able to decide if the results should be larger or smaller than the givens

SPH4U: Relativity and Energy

Recorder: _____
Manager: _____
Speaker: _____
0 1 2 3 4 5

The consequences of Einstein's bold suggestion, that the speed of light is constant for all inertial reference frames, go far beyond just space and time – they also extend to our notions of energy. Using a clever argument, Einstein created the world's most famous equation:

$$E = \gamma mc^2 \text{ where } \gamma = (1-v^2/c^2)^{-½}$$

This is usually written, for the general public, as $E_o = mc^2$, where the "$_o$" is carelessly left out! Sometimes physics ideas stretch beyond our common sense and we begin to rely on equations to help us understand how our universe works. Let's explore this equation and try to figure out what it tells us about energy.

A: The Mass-Energy Relationship

1. **Reason.** Describe carefully how this energy depends on the speed of an object.

2. **Reason.** What other type of energy depends on an object's speed? What does this tell us about the type of energy Einstein's equation describes?

3. **Reason.** According to the equation, how much energy does an object have when it is at rest? Explain how the equation for E becomes the equation for E_o. Is Einstein's equation still describing kinetic energy?

4. **Reason.** When at rest, what is the only characteristic of the object that could be changed and affect the amount of energy E_o? What does this suggest about where this energy might be stored?

An object at rest possesses a form of energy called its *rest energy*, E_o, Einstein's complete expression ($E = \gamma mc^2$) gives the **total energy** of the object, which always includes the rest energy and possibly some kinetic energy depending on the object's velocity. To the best of our knowledge, this equation is correct under all circumstances and replaces the ones we have previously learned.

5. **Represent.** Write an expression that shows the relationship between E, E_o *and* E_k.

*** check this with your teacher before moving on ***

B: Relativistic Energy

1. **Calculate.** Consider a 1.0 kg block initially at rest. It experiences a force that eventually causes it to reach an impressive speed of 0.6 c. Imagine we had learned nothing about relativity - determine the energies for the "Before Einstein" column in the chart below. Use Einstein's equation to help find the energies for the "After Einstein" column.

	Before Einstein (B. E.)	After Einstein (A. E.)
Rest Energy	zero	
Kinetic Energy		
Total Energy		

2. **Explain.** How do you use Einstein's equation to find the kinetic energy?

3. **Reason.** Under what condition is the expression $\frac{1}{2}mv^2$ valid? What should we conclude about the limitations of the traditional kinetic energy equation?

Accelerating an object to speeds near that of light is extremely challenging and with our current technology, we can only accomplish this for atoms and sub-atomic particles. According to Newton, all we need to do is exert a steady force on something for long enough and the uniform acceleration will eventually cause the object to reach 3.0×10^8 m/s and our science fiction dreams will come true. According to Einstein, things are different.

4. **Reason.** How much energy is required to bring the 1.0 kg block to the speed of light? Explain the mathematical difficulty with performing this calculation. Explain to your kid sister how much energy would you "need".

5. **Reason.** What does the difficulty of the previous calculation imply about the possibility of ever reaching or exceeding the speed of light?

This is the main reason why the latest and greatest particle accelerator, the Large Hadron Collider ($ 9 000 000 000), is such a colossal engineering feat. A tremendous amount of energy is required to accelerate the collider's protons to 0.999 999 991 c.

C: Particle Physics

1. **Calculate.** A proton is a very small particle with a mass of 1.673×10^{-27} kg. How much energy is stored in the mass of the particle?

Subatomic particles usually possess very small quantities of energy. A new unit is needed to conveniently notate these small values. One *electron volt* (eV) is a unit of energy equivalent to 1.602×10^{-19} J.

2. **Calculate.** Find the proton's rest mass energy in terms of MeV (Mega eV).

3. **Explain.** Physicists often write the mass of the proton as 938.3 MeV/c^2. Use the rest-energy equation to help explain why this is a valid unit for mass. Explain why these units make it easy to calculate the rest energy.

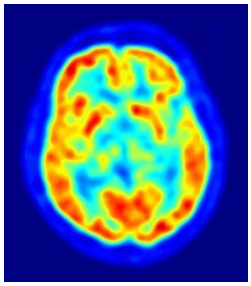
Is it possible to release the energy stored in a particle's mass? You may have already studied the process of nuclear fusion or fission in another course and have learned that, yes, this is possible. In a typical fusion reaction (like in the sun), a deuterium particle (1876 MeV/c^2) fuse with a tritium particle (2809 MeV/c^2)producing a helium nuclei (3729 MeV/c^2) and a neutron (937 MeV/c^2).

$$D + T \rightarrow He + n$$

4. **Reason.** How does the mass of the reactants compare with the mass of the products? What happened to the mass? What does this imply about the conservation of mass? Speculate on a new, better conservation law.

5. **Calculate.** How much energy is released during the fusion process? Give your answer in joules and electron volts.

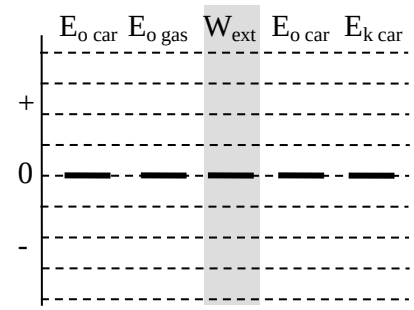
The conversion of matter to energy can be total if a matter particle collides with a corresponding anti-matter particle. This is the *raison d'etre* of the Large Hadron Collider: to collide protons and anti-protons, which releases a tremendous amount of energy. This is also the physics behind the medical imaging technique positron imaging tomography (PET scans), where an electron collides with a positron (the anti-electron). In the case of the PET scan, radioactive materials are injected into the blood stream of a patient. The decay process releases a positron (an anti-electron) which collides with an electron of a nearby atom. In the process, the two particles annihilate and produce two gamma-ray photons (γ).



$$e^- + e^+ \rightarrow \text{energy (two photons)}$$

6. **Calculate.** How much energy is released when an electron (0.511 MeV/c^2) collides with a positron (same mass) and the two annihilate (leave no mass behind)? You may assume they are both essentially at rest.

7. **Calculate.** In Star Trek, the main power source for the starship Enterprise is a matter-antimatter engine. How much energy would be produced by annihilating 1.0 L of gasoline (0.720 kg) with 1.0 L of anti-gasoline (0.720 kg)? What form of energy is the annihilation energy transformed into? What speed would that accelerate a typical car (1200 kg) to (use $\frac{1}{2}mv^2$)?



8. **Calculate.** The previous result is quite fast! We should confirm this with a more reliable calculation using Einstein's equation to solve for γ and then *v*. Use the result for γ explain why the result from #7 was reasonably accurate.

9. **Calculate.** The reverse process can also take place! Energy can be converted into a particle – antiparticle pair.

$$e^- + e^+ \rightarrow p + \overline{p}$$

In this case, the extra kinetic energy of the electron-positron pair is converted into the mass of the proton and anti-proton. This is exactly what used to happen at the LEP (Large Electron Positron) collider at CERN in Switzerland. What should the speed be of an electron and positron in the LEP to allow this to happen such that the proton – anti-proton pair are created at rest?
(Hint: make a bar chart, use it to find γ, then use γ to find speed)